Experimental Investigation of Continuous Variable Entanglement with Spatial Degrees of Freedom of Photons

Stephen Walborn

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UFF, setembro 2009

Summary

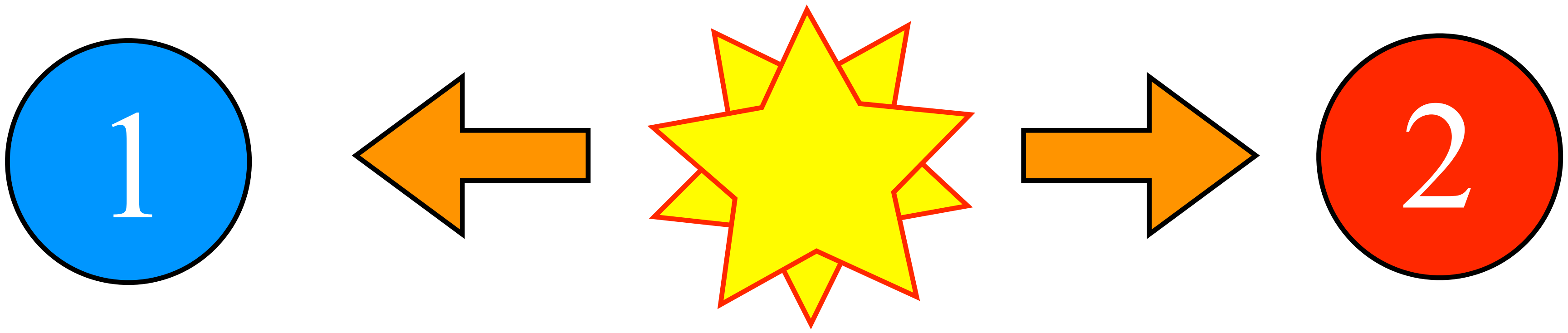
- Quantum Entanglement
- Spatial Entanglement with Spontaneous Parametric Down-conversion
- Gaussian vs. Non-Gaussian states
- Detecting Genuine Non-Gaussian Entanglement
- EPR Non-locality
- Non-local Optical Vortex

Entanglement

Entanglement

A Quantum Correlation

Ex: decay process



$$|\Psi\rangle_{12} \neq |\psi\rangle_1 |\phi\rangle_2$$

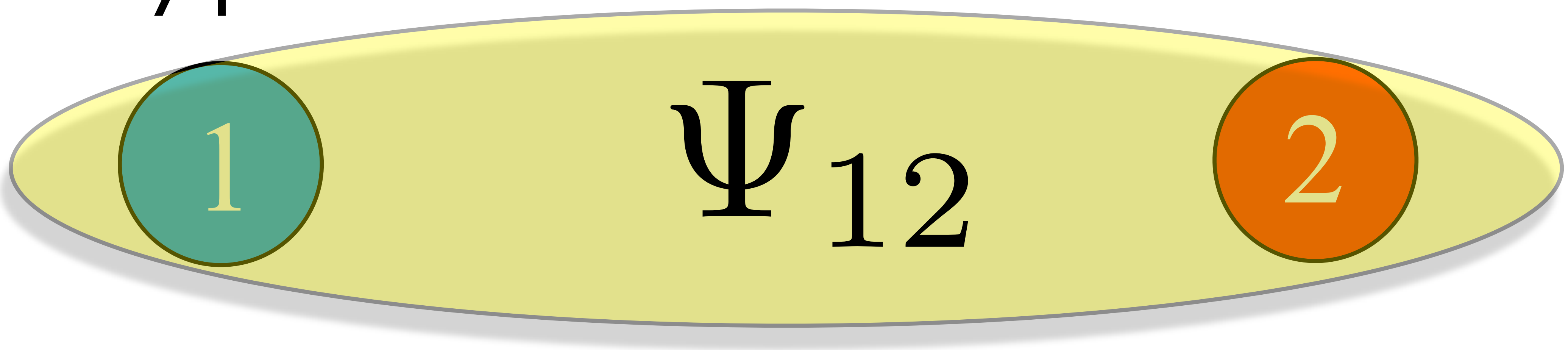
Ex:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2\right)$$

Entanglement

A Quantum Correlation

Ex: decay process



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Ex:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}} \left(|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2 \right)$$

Why is Entanglement Interesting?

- Fundamental difference between quantum and classical physics
- Resource for quantum information tasks, e.g. teleportation, quantum computing, key distribution, etc
- Present in laboratories! (Ions, photons, atoms, intense beams, ...)

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}} \left(|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2 \right)$$

Continuous Variable Entanglement

$$|\Psi\rangle_{12} \neq |\psi\rangle_1 |\phi\rangle_2$$

Wave-function

$$\Psi(x_1, x_2) \neq \psi(x_1)\phi(x_2)$$

Ex: Einstein-Podolsky-Rosen state

$$|EPR\rangle = \int \delta(x_1 - x_2)|x_1\rangle|x_2\rangle dx_1 dx_2$$

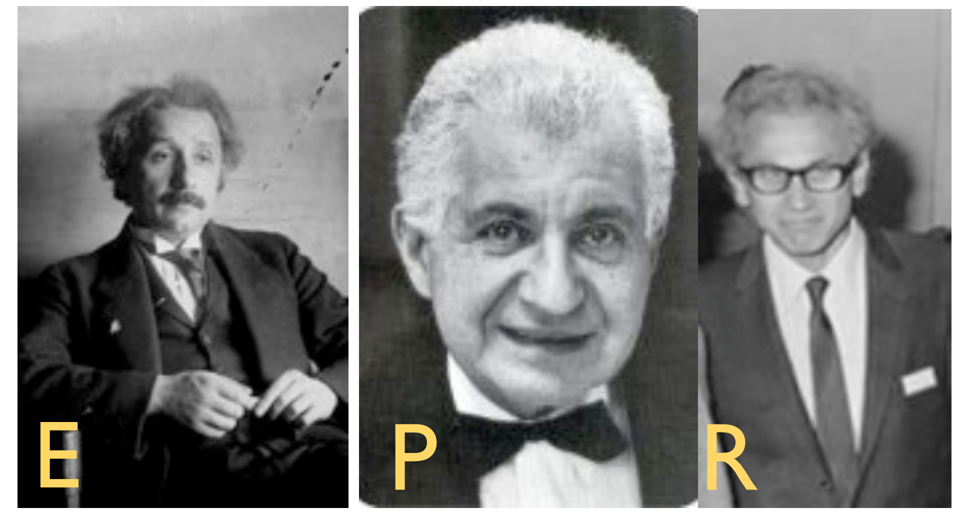
$$= \int \delta(p_1 + p_2)|p_1\rangle|p_2\rangle dp_1 dp_2$$

$p_1 + p_2$ and $x_1 - x_2$ well-defined

x_1, x_2, p_1, p_2 are UNdefined
(all values equiprobable)

EPR paradox (1935)

Considerations

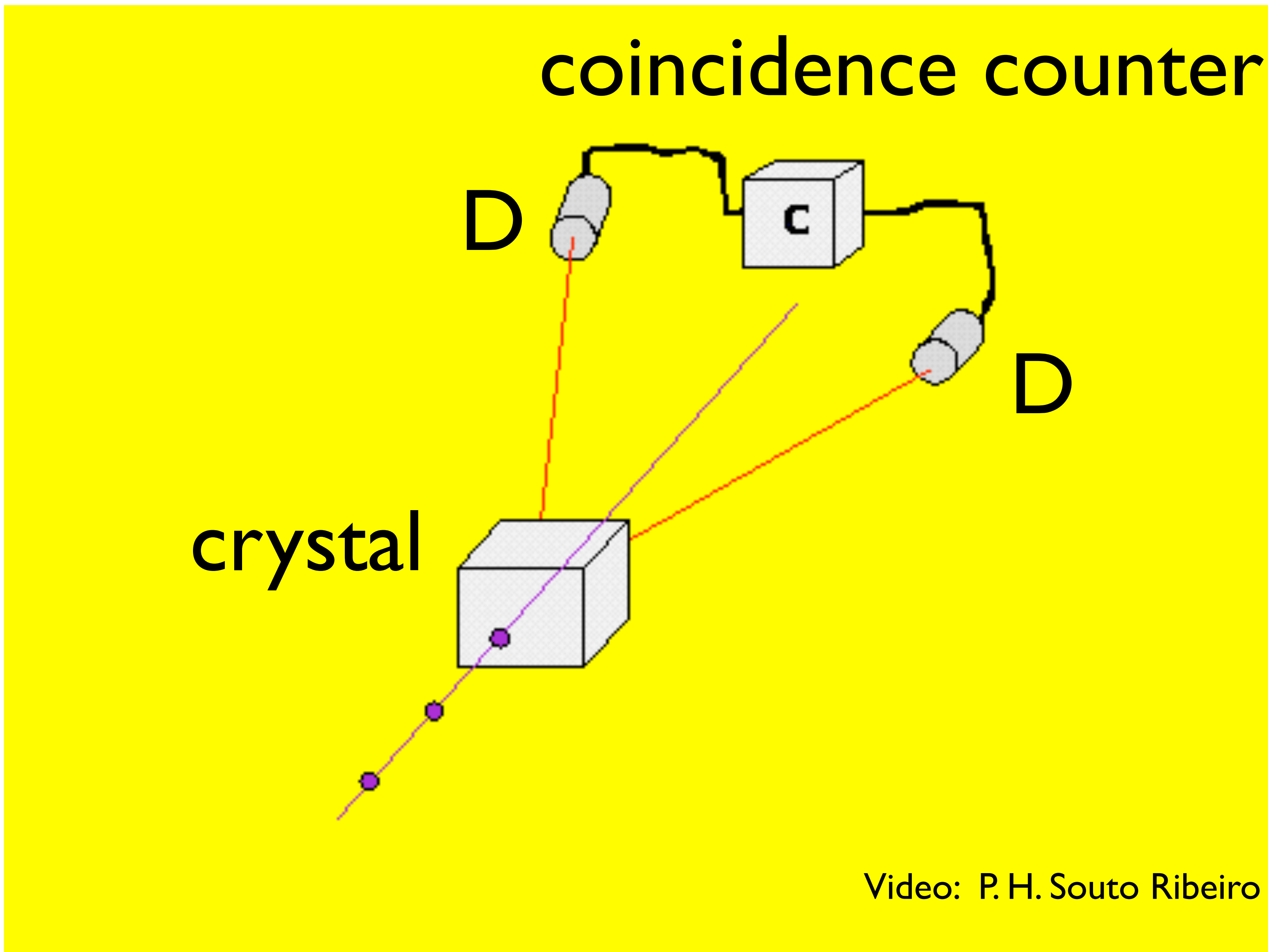


- Measurement x_1 or p_1 completely determines x_2 or p_2
- systems 1 and 2 spatially separate, we are free to chose which measurement (x_1 or p_1)
- this implies that x_2 and p_2 are predetermined, but x_2 and p_2 complementary observables?

⇒ Quantum Mechanics is incomplete

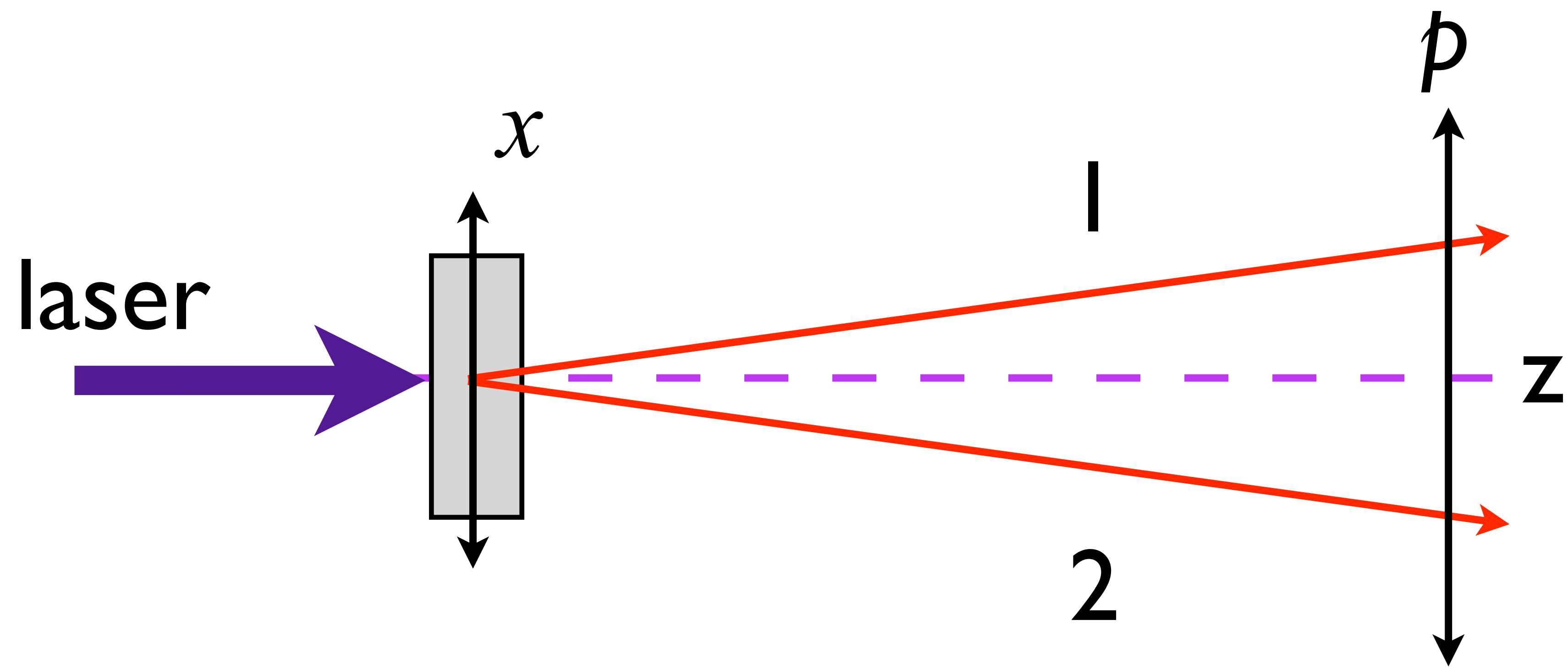
Spontaneous Parametric Down Conversion and Spatial Entanglement

Spontaneous Parametric Down Conversion (SPDC)



Coincidence counting isolates two-photon events

Transverse Spatial Entanglement in SPDC



Photons "born" at same point in crystal

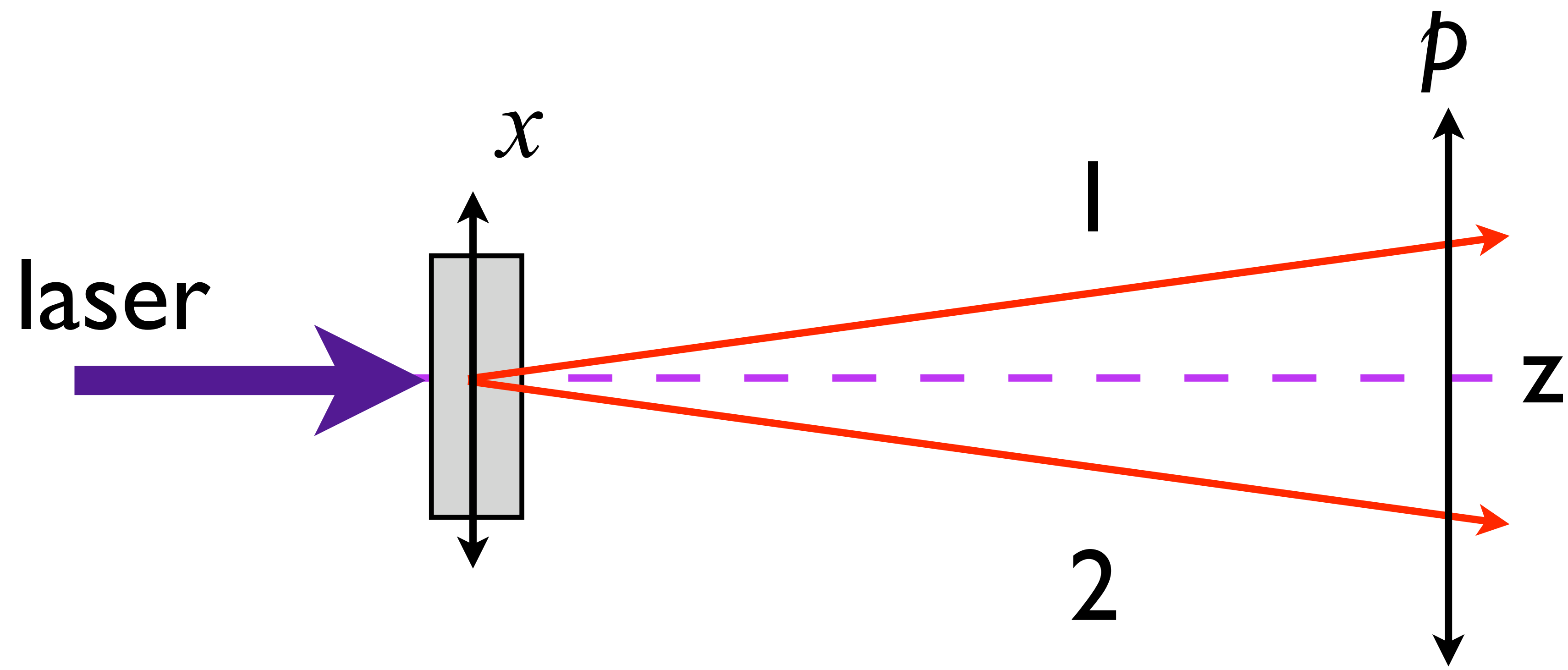
position (near field) correlation:

$$x_1 - x_2 = 0$$

Conservation of Transverse Momentum: far field correlation

$$p_1 + p_2 = p$$

Transverse Spatial Entanglement in SPDC



Photons “born” at same point in crystal

position (near field) correlation:

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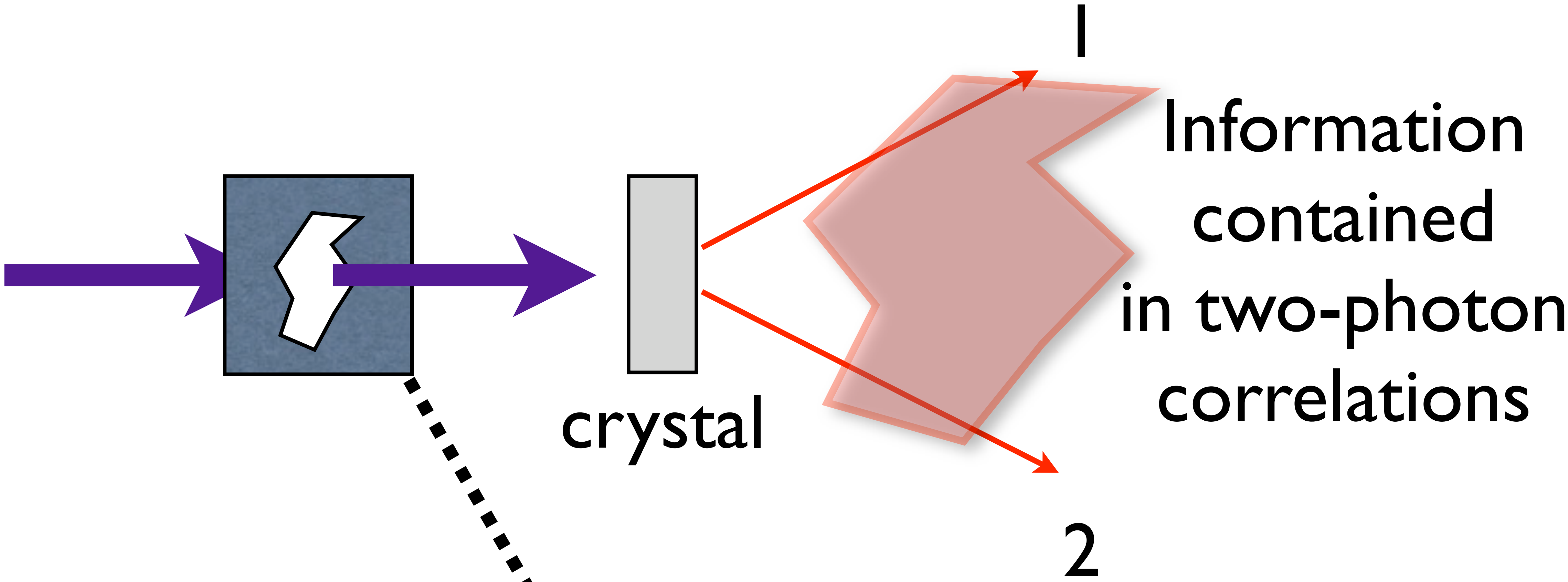
Conservation of Transverse Momentum: far field correlation

$$p_1 + p_2 = p$$

Highly correlated quantum states (ENTANGLEMENT)

Producing spatial entanglement



$$|\psi\rangle = \iint d\boldsymbol{p}_1\boldsymbol{p}_2 v(\boldsymbol{p}_1 + \boldsymbol{p}_2)\gamma(\boldsymbol{p}_1 - \boldsymbol{p}_2)|\boldsymbol{p}_1\rangle|\boldsymbol{p}_2\rangle$$

Entanglement depends on form of functions v and ϒ

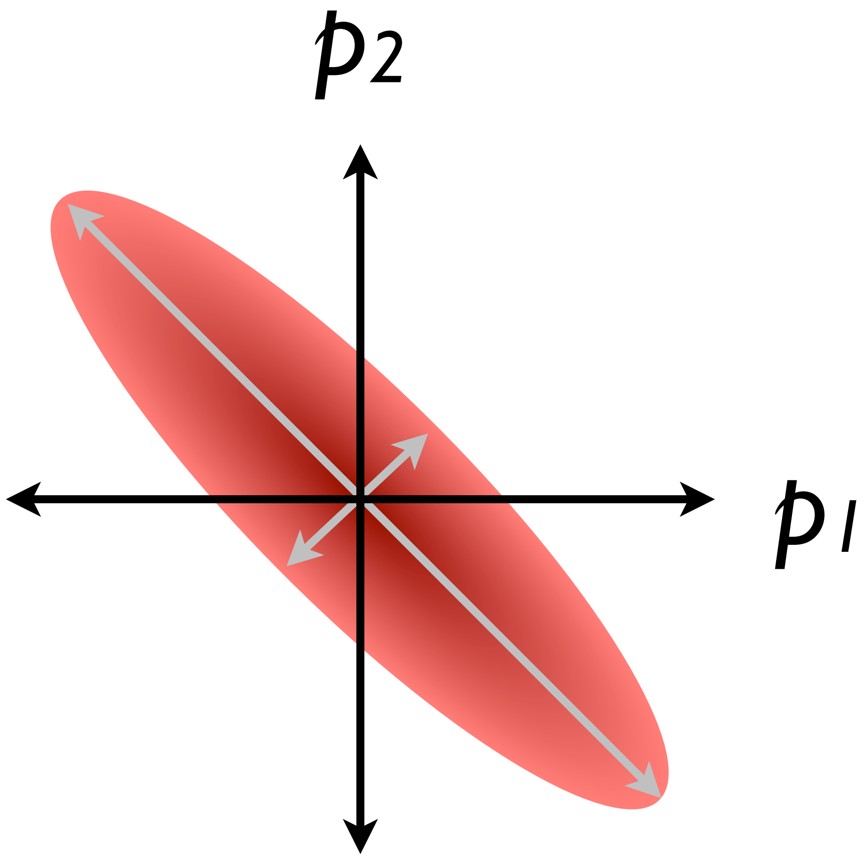
Monken , Pádua, Souto Ribeiro, PRA 57 3123 (1998).

SPDC state* ➡ "EPR" state

$$|\psi\rangle = \iint dp_1 p_2 v(p_1 + p_2)\gamma(p_1 - p_2)|p_1\rangle|p_2\rangle$$

v(p) is narrow gaussian

ϒ*(p)* is wide gaussian



Pump is "plane wave"

Approximates EPR state

(*from now on, we'll work in 1-D)

Detecting Entanglement

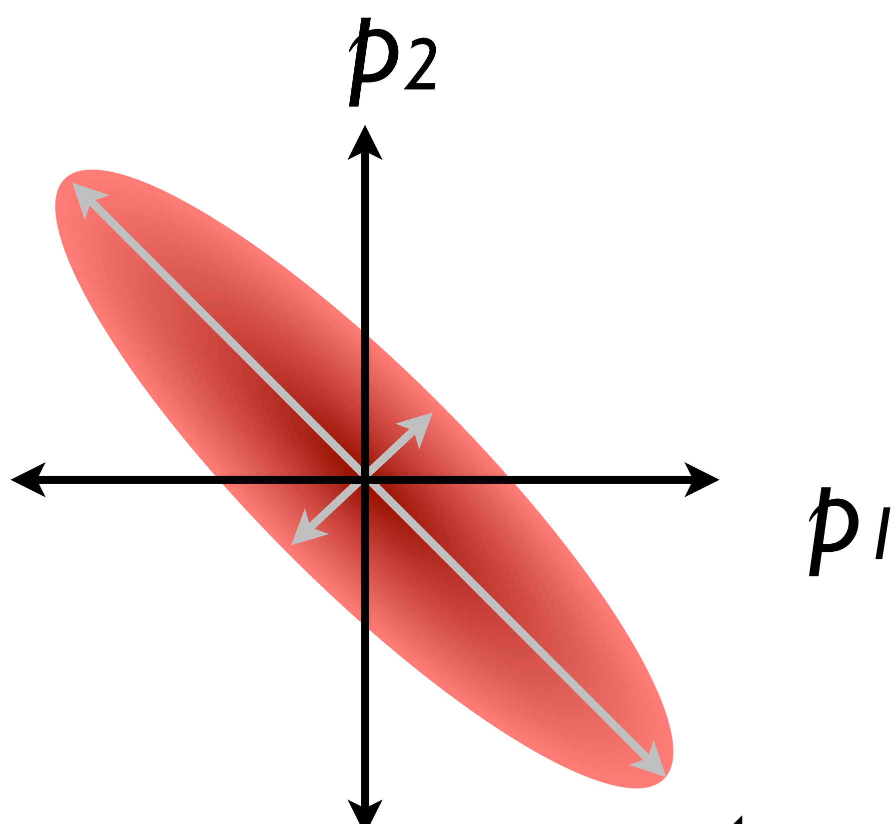
(Second-order) variance inequalities

Separable if: $$(\Delta x_-)^2(\Delta p_+)^2 \geq 1$$

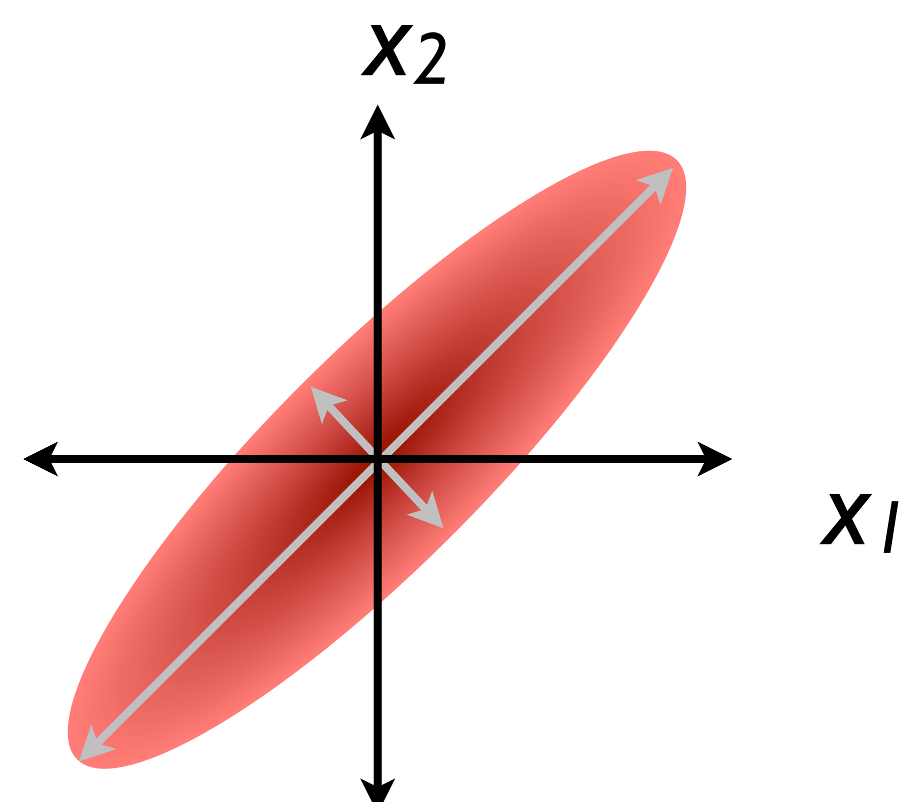
Mancini et al. [PRL 88 120401 (2002)]

$p_+ = p_1 + p_2$

$x_- = x_1 - x_2$



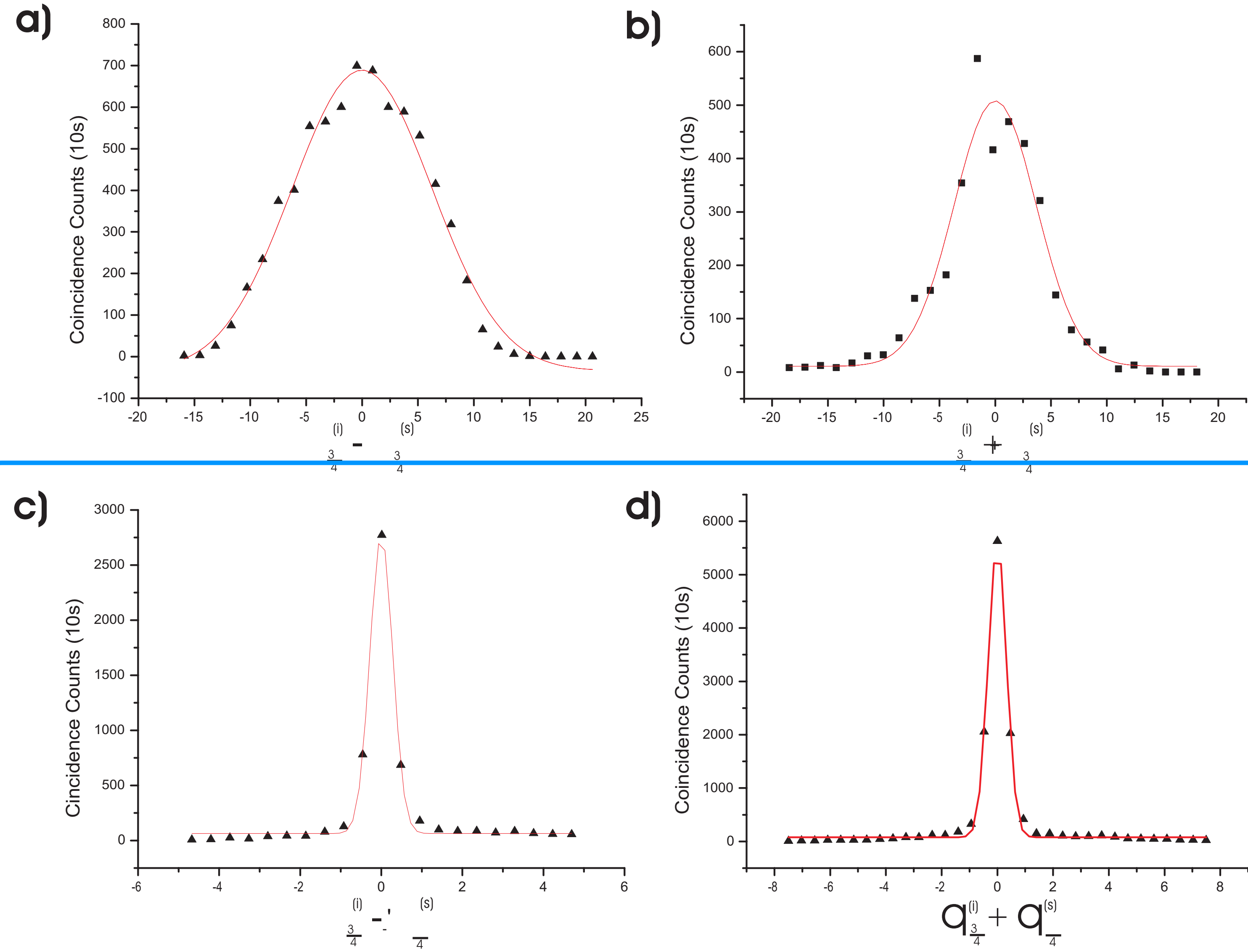
transverse momentum
(far-field, Fourier)



transverse position
(near field, image)

Typical Experimental Results

$$(\Delta x_+)^2 = 13.6 \qquad (\Delta p_-)^2 = 39.1$$



$$(\Delta x_-)^2 = 0.1 \qquad (\Delta p_+)^2 = 0.1$$

violation of variance inequalities

Tasca, Walborn, Toscano, Souto Ribeiro, PRA (2008).

Why Study Spatial Entanglement?

- Explore Higher-dimensional Hilbert space
- Possible interest for quantum info tasks (Cryptography, better Bell-inequality violation, etc.)
- Easy and robust production/manipulation of quantum state
- Continuous-variable or discrete bases
- Produce high-quality, entangled pure states (coincidence counts)

Gaussian vs. Non-gaussian States

Gaussian States

- Wave-function, Wigner functions, etc are gaussians
- All correlation functions reduce to first or second order (few parameters)
- All information contained in Covariance Matrix (of second-order moments)
- Second-order variance criteria identify entanglement

Non-Gaussian states

- Higher-order information required, more complex
- Non-Gaussian states (or operations) are necessary for Bell's inequality violation, universal quantum computing, entanglement distillation,...

Genuine Non-Gaussian Entanglement: Entanglement which is invisible to any second-order (variance) criteria

Detecting Entanglemment: Considerations

- Second-order inequalities are necessary and sufficient conditions only for Gaussian states
- Second-order (variance) inequalities may fail to identify "non-Gaussian" entanglement
- Higher-order entanglement criteria exist which identify non-Gaussian entanglement

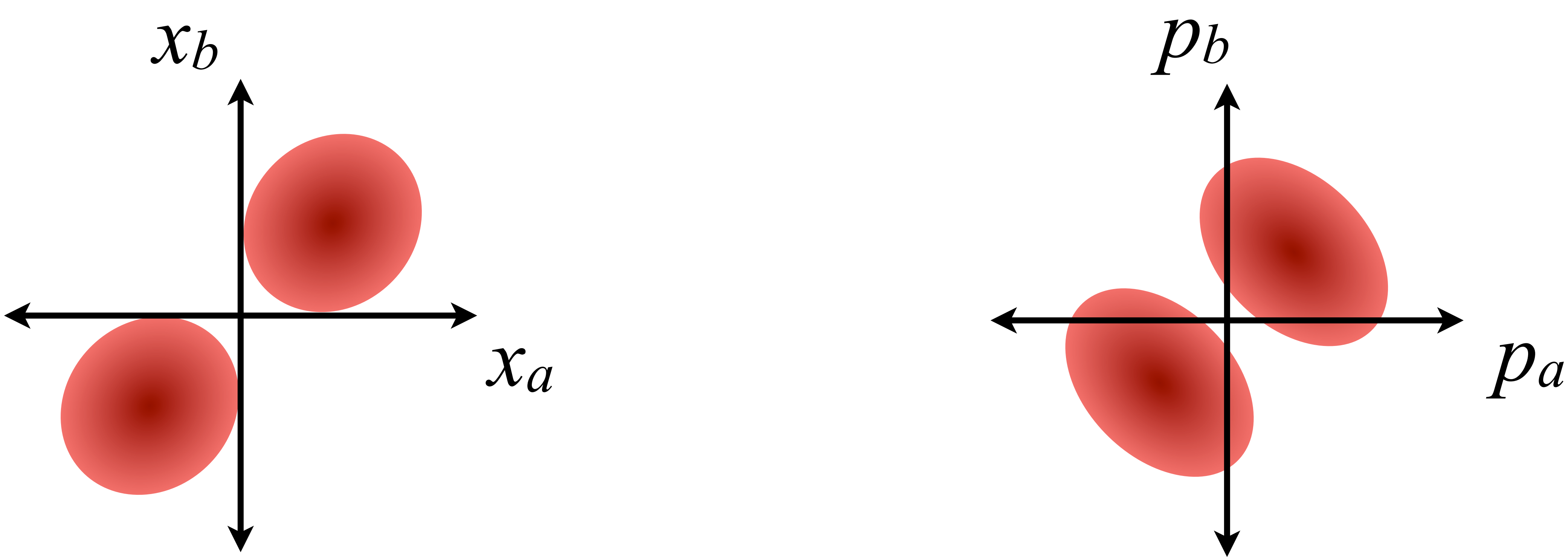
Detecting Genuine Non-Gaussian Entanglement

State with "Genuine Non-Gaussian Entanglement"

$$\Psi(x_+, x_-) = N \boxed{x_+ \exp(-x_+^2/4s^2)} \exp(-x_-^2/4t^2)$$

$$x_\pm = (x_a \pm x_b)/2$$

Hermite-Gauss function



Genuine non-gaussian entanglement if*:

$$\langle \Delta\hat{x}_a \Delta\hat{x}_b \rangle \langle \Delta\hat{p}_a \Delta\hat{p}_b \rangle - \langle \Delta\hat{x}_a \Delta\hat{p}_b \rangle \langle \Delta\hat{p}_a \Delta\hat{x}_b \rangle \geq 0$$

This is satisfied when $0.57 < s/t < 1.73$

**R. Simon, PRL (2000)*

Shchukin-Vogel Criteria

Criteria to test if state is entangled (PPT)

An example, D_{HO} =

$$1 + r^4\langle\Delta^2(\hat{x}_a\hat{x}_b)\rangle + \langle\Delta^2(\hat{x}_a\hat{p}_b)\rangle + \langle\Delta^2(\hat{p}_a\hat{x}_b)\rangle$$
$$+\frac{1}{r^4}\langle\Delta^2(\hat{p}_a\hat{p}_b)\rangle + 2\langle\hat{x}_a\hat{p}_b\rangle\langle\hat{p}_a\hat{x}_b\rangle - 2\langle\hat{x}_a\hat{x}_b\rangle\langle\hat{p}_a\hat{p}_b\rangle$$
$$-r^2[\langle\hat{x}_a^2\rangle + \langle\hat{x}_b^2\rangle] - \frac{\langle\hat{p}_a^2)\rangle+\langle\hat{p}_b^2\rangle}{r^2} \geq 0$$

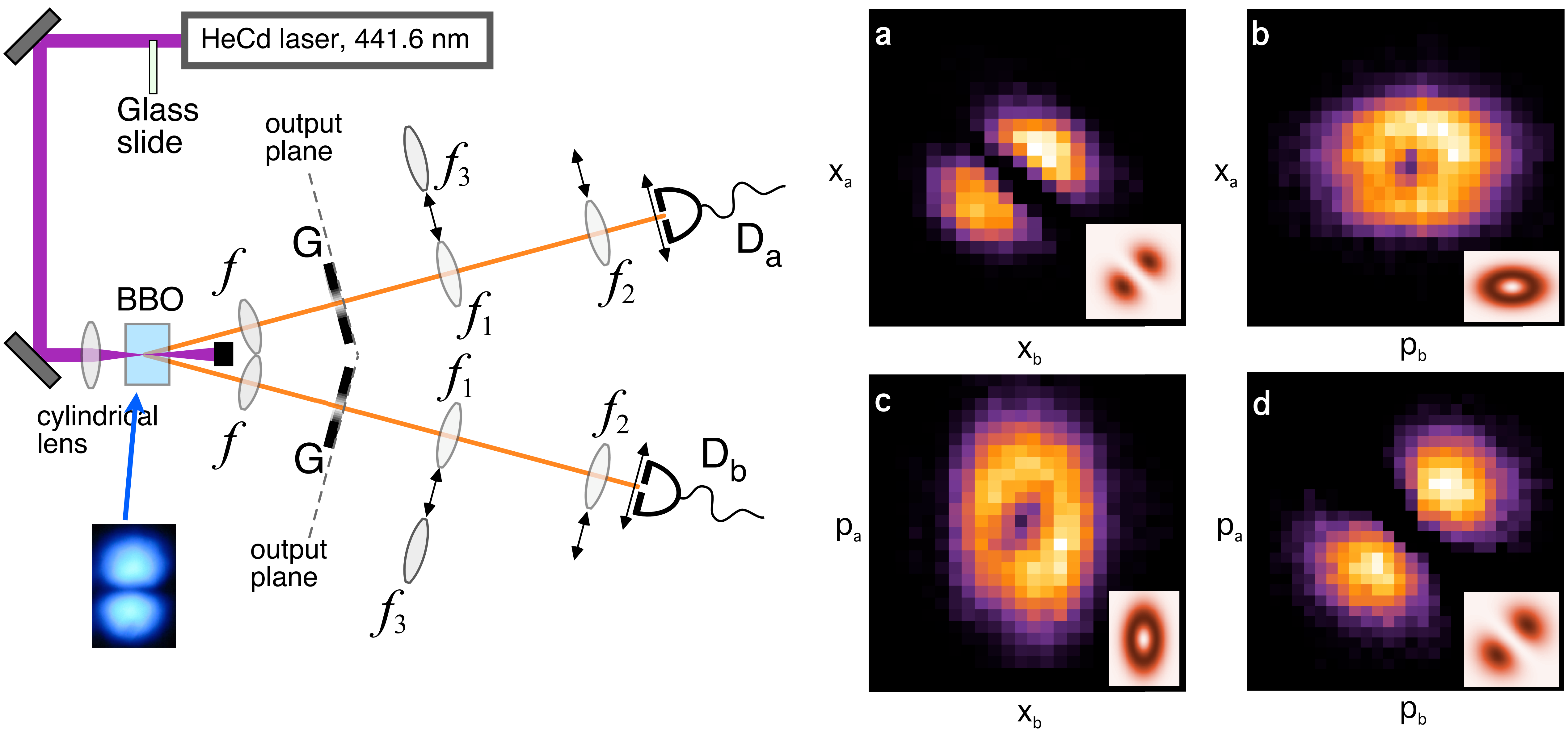
r = local scaling factor

If $D_{HO} < 0$, state is entangled

$D_{HO}<0$ provided $0.63 < s/t < 1.58$

Shchukin and Vogel, PRL (2005).

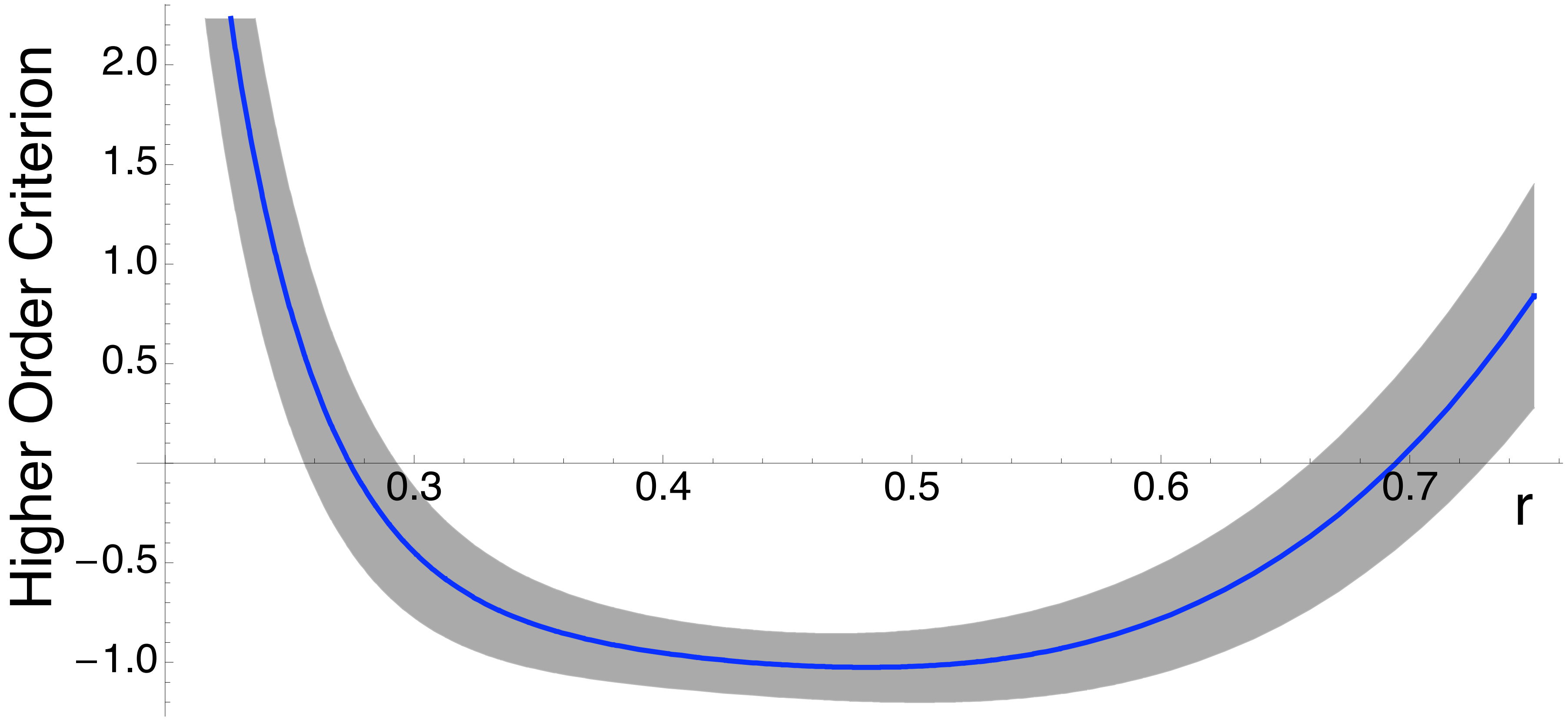
Experimental Test



$$\langle \hat{w}_a^n \hat{w}_b^m \rangle = \sum_{j,k} w_{aj}^n w_{bk}^m P(w_{aj}, w_{bk})$$

Experimental Results

D_{HO}



local scaling factor

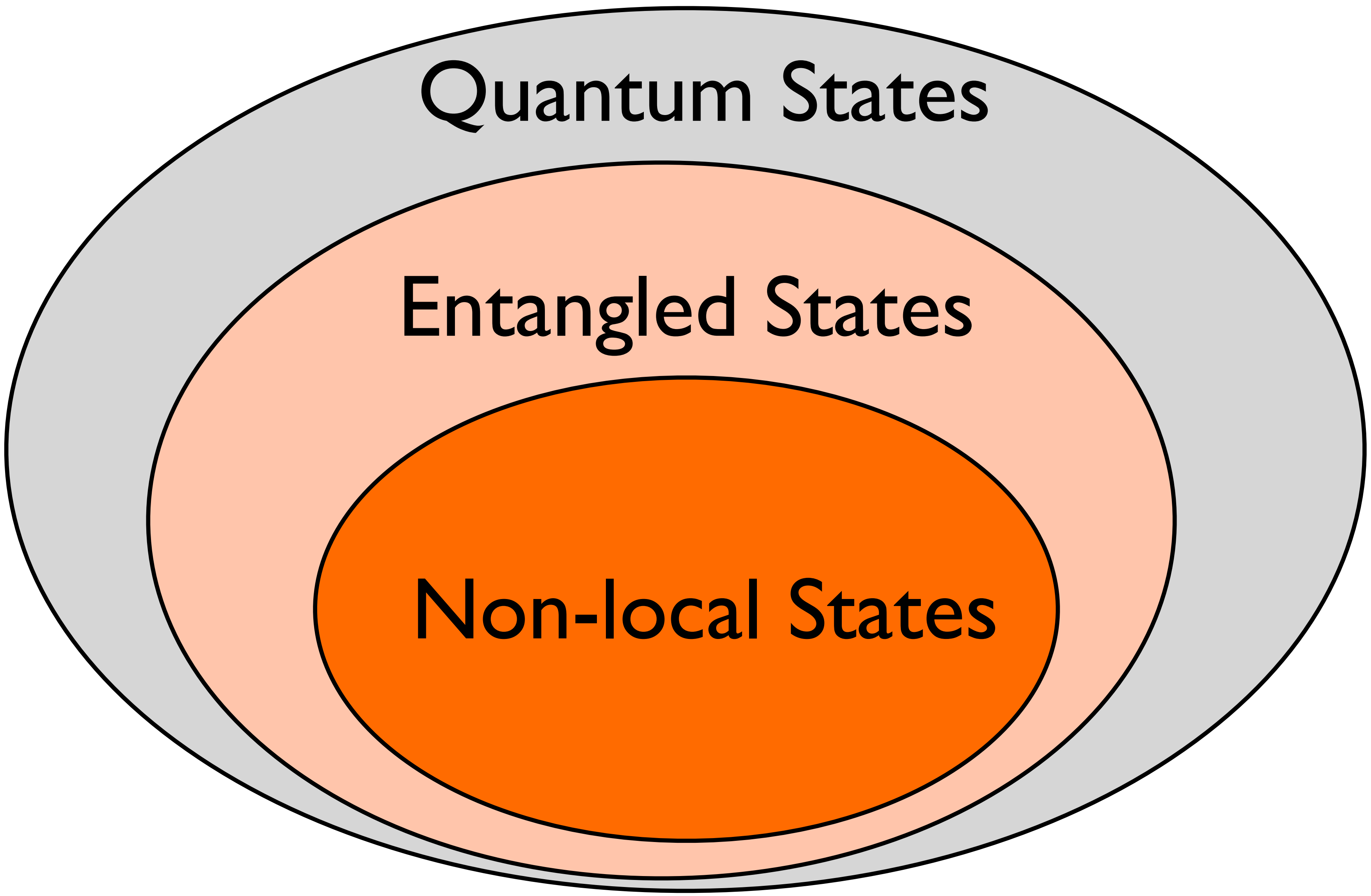
$$1 + r^4\langle\Delta^2(\hat{x}_a\hat{x}_b)\rangle + \langle\Delta^2(\hat{x}_a\hat{p}_b)\rangle + \langle\Delta^2(\hat{p}_a\hat{x}_b)\rangle$$
$$+\frac{1}{r^4}\langle\Delta^2(\hat{p}_a\hat{p}_b)\rangle + 2\langle\hat{x}_a\hat{p}_b\rangle\langle\hat{p}_a\hat{x}_b\rangle - 2\langle\hat{x}_a\hat{x}_b\rangle\langle\hat{p}_a\hat{p}_b\rangle$$
$$-r^2[\langle\hat{x}_a^2\rangle + \langle\hat{x}_b^2\rangle] - \frac{\langle\hat{p}_a^2)\rangle+\langle\hat{p}_b^2\rangle}{r^2} \geq 0$$

Second-order test = 0.39 > 0

Gomes, Salles, Toscano, Souto Ribeiro, Walborn, submitted

EPR Non-locality with Non-gaussian states

Non-locality vs. Entanglement



Can we show that our Non-gaussian state is non-local?

EPR non-locality

Reid EPR inequality

$$\Delta^2(x_a|x_b)\Delta^2(p_a|p_b) \geq \frac{1}{4}$$

Violation indicates a situation in which the EPR argument is valid

For our non-gaussian state:

$$\Delta^2(x_a|x_b)\Delta^2(p_a|p_b) \approx 0.45 \pm 0.01$$

No EPR non-locality?

Entropic EPR inequality*

$$h(X_A|X_B) + h(P_A|P_B) \geq \ln \pi e$$

conditional Shannon entropy

For our non-gaussian state:

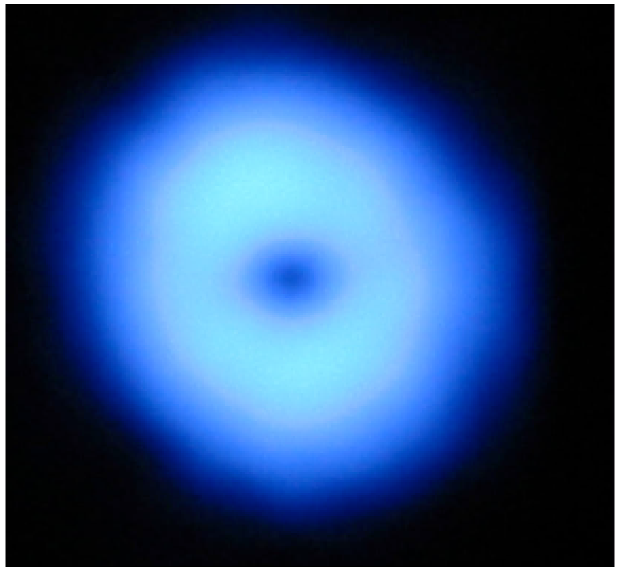
$$h(X_A|X_B) + h(P_A|P_B) \approx 1.95 \pm 0.04 < \ln \pi e$$

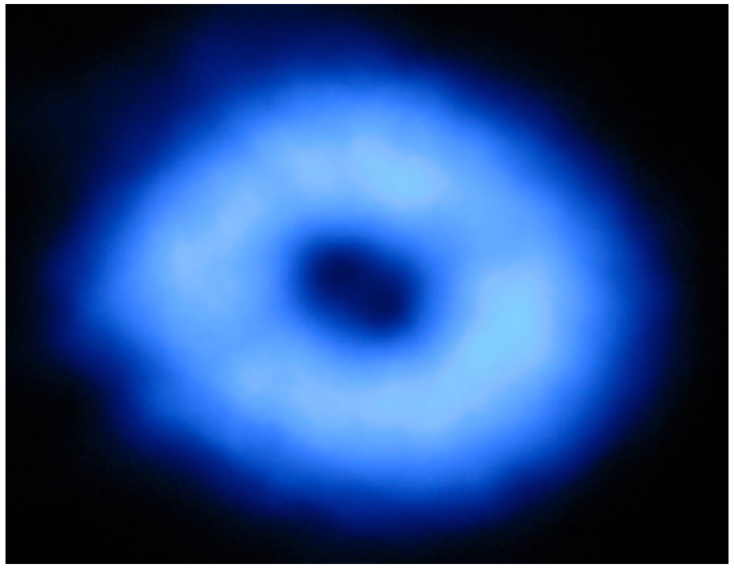
⇒ EPR non-local

* SPW, Gomes, Salles, Toscano, Souto Ribeiro, arXiv:0907.4263

Non-local Optical Vortex

Laguerre-Gaussian Modes





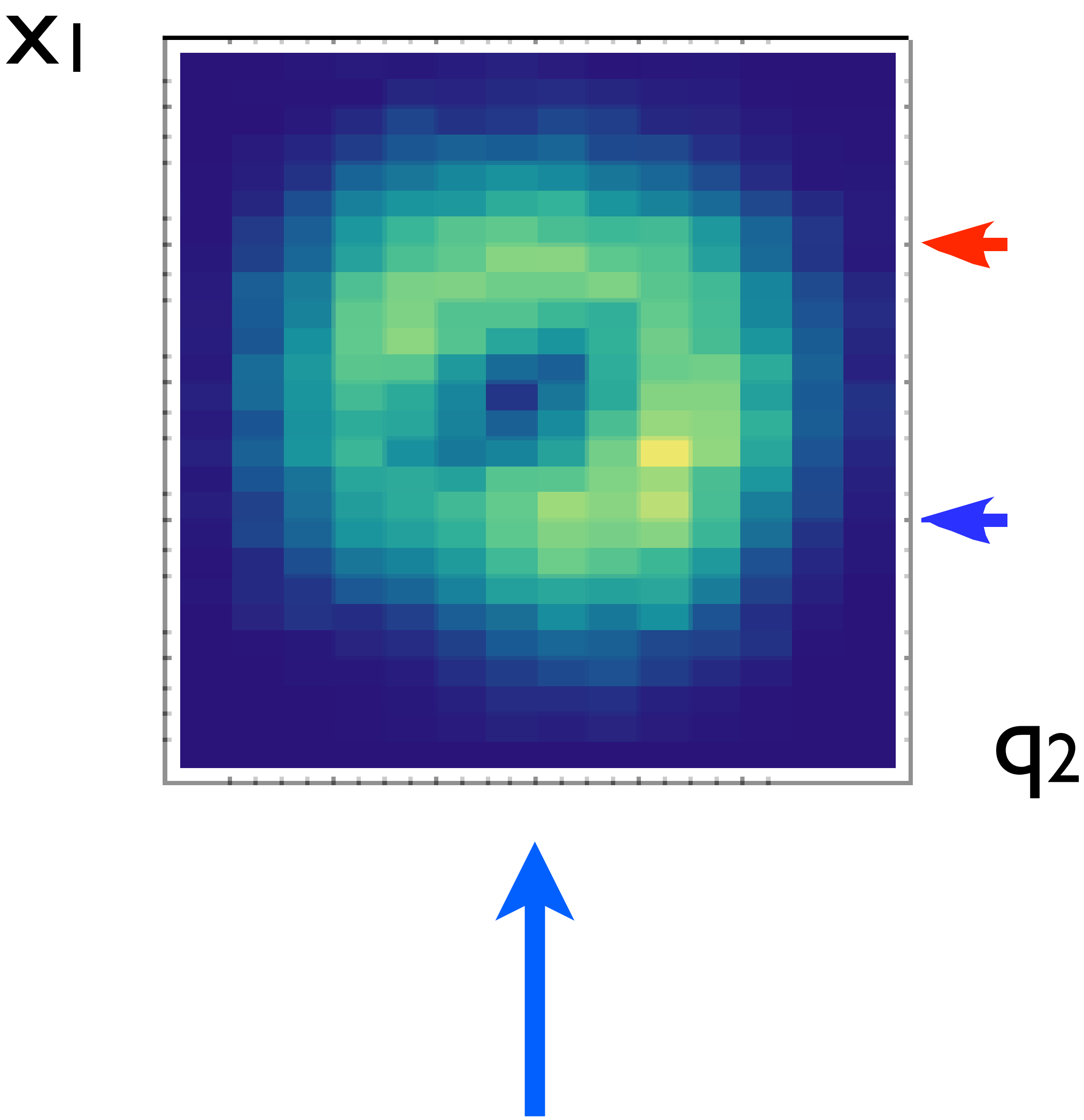
LG_{01} LG_{02}

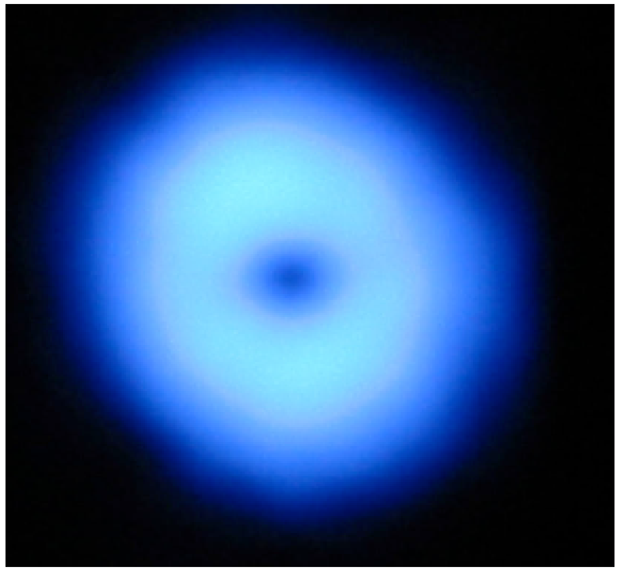
Carry orbital angular momentum: optical vortices

Zero-intensity region corresponds to phase singularity

photos: Monken Lab, UFMG

Results Revisited





Laguerre Gauss Beam

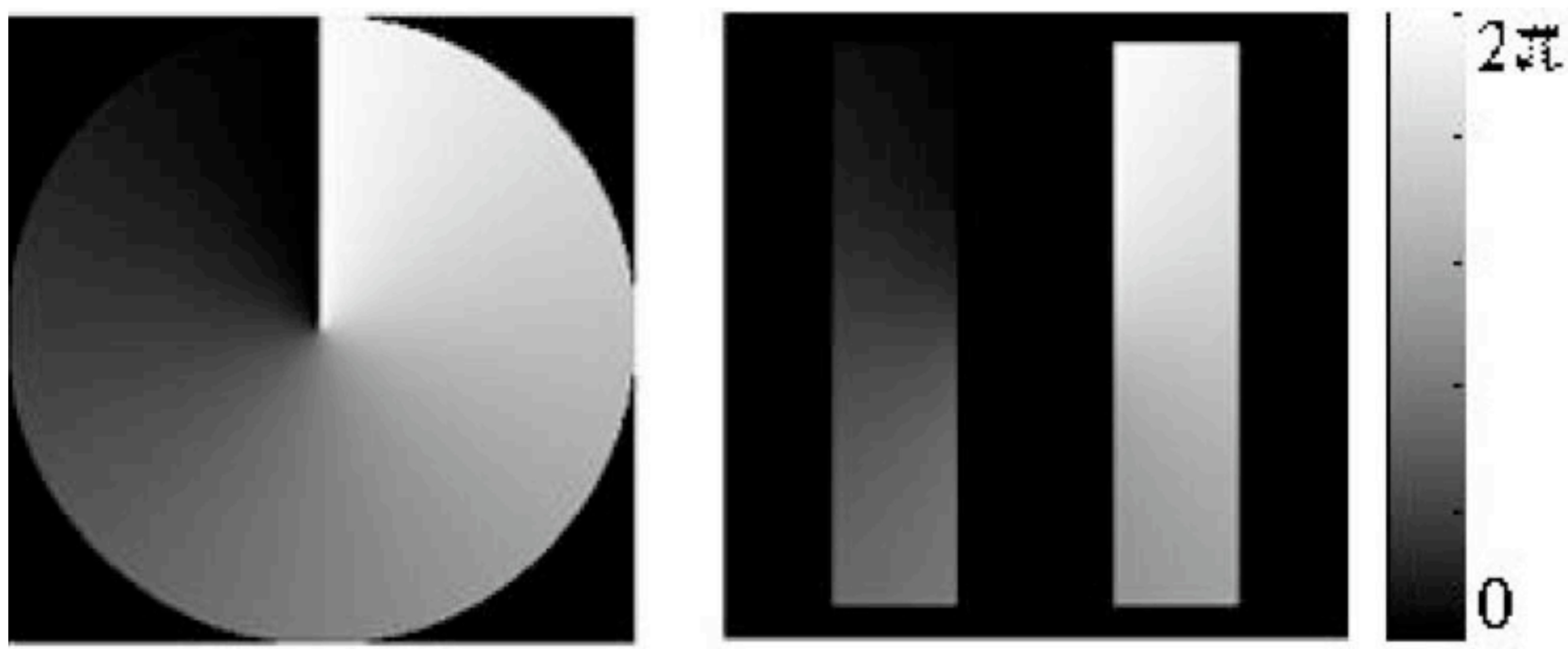
Laguerre-Gaussian profile?

How to measure phase dependence?

Interferometry

Classical LG beam at a double slit

azimuthal phase dependence

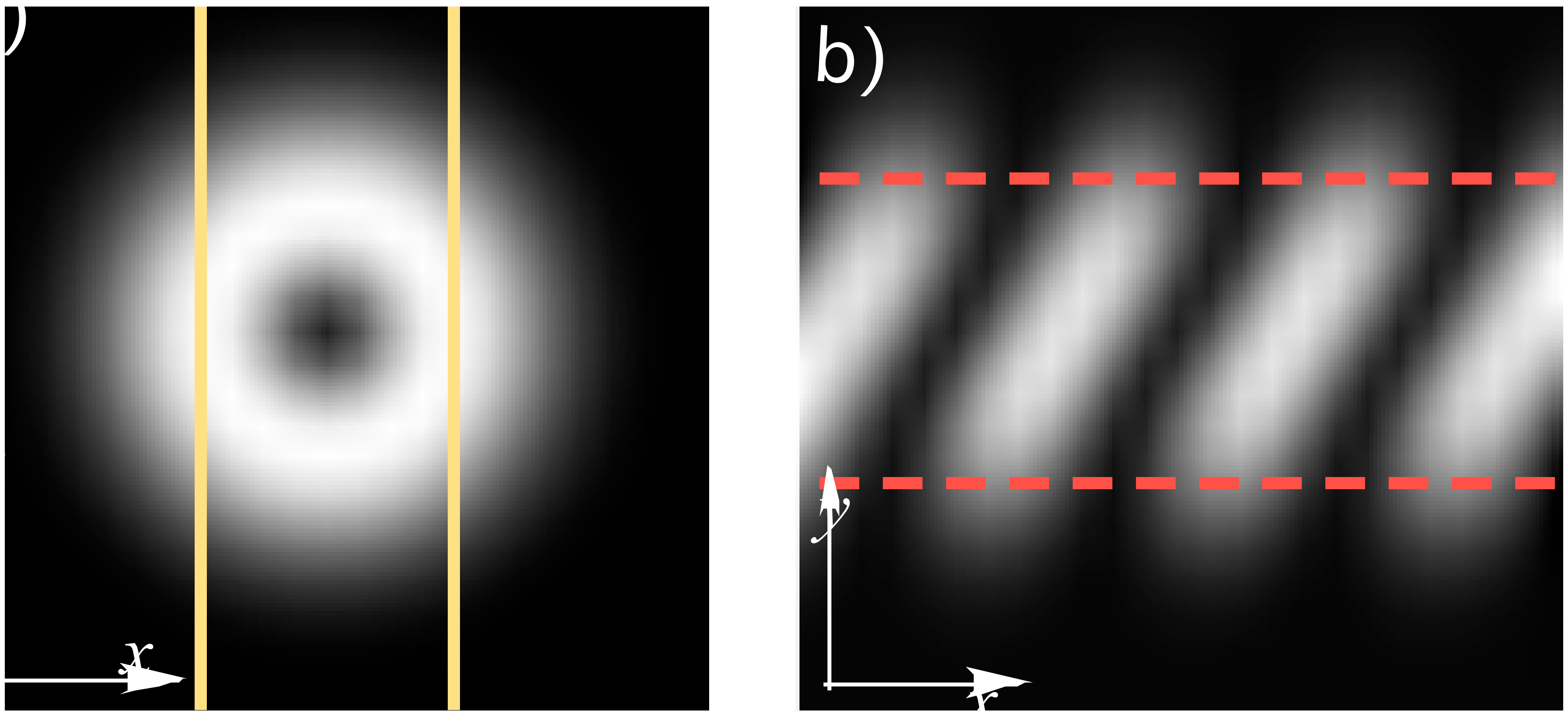


slit 1 slit 2

Young's interference depends upon azimuthal phase

Sztul and Alfano, Optics Letters 31 999 (2006)

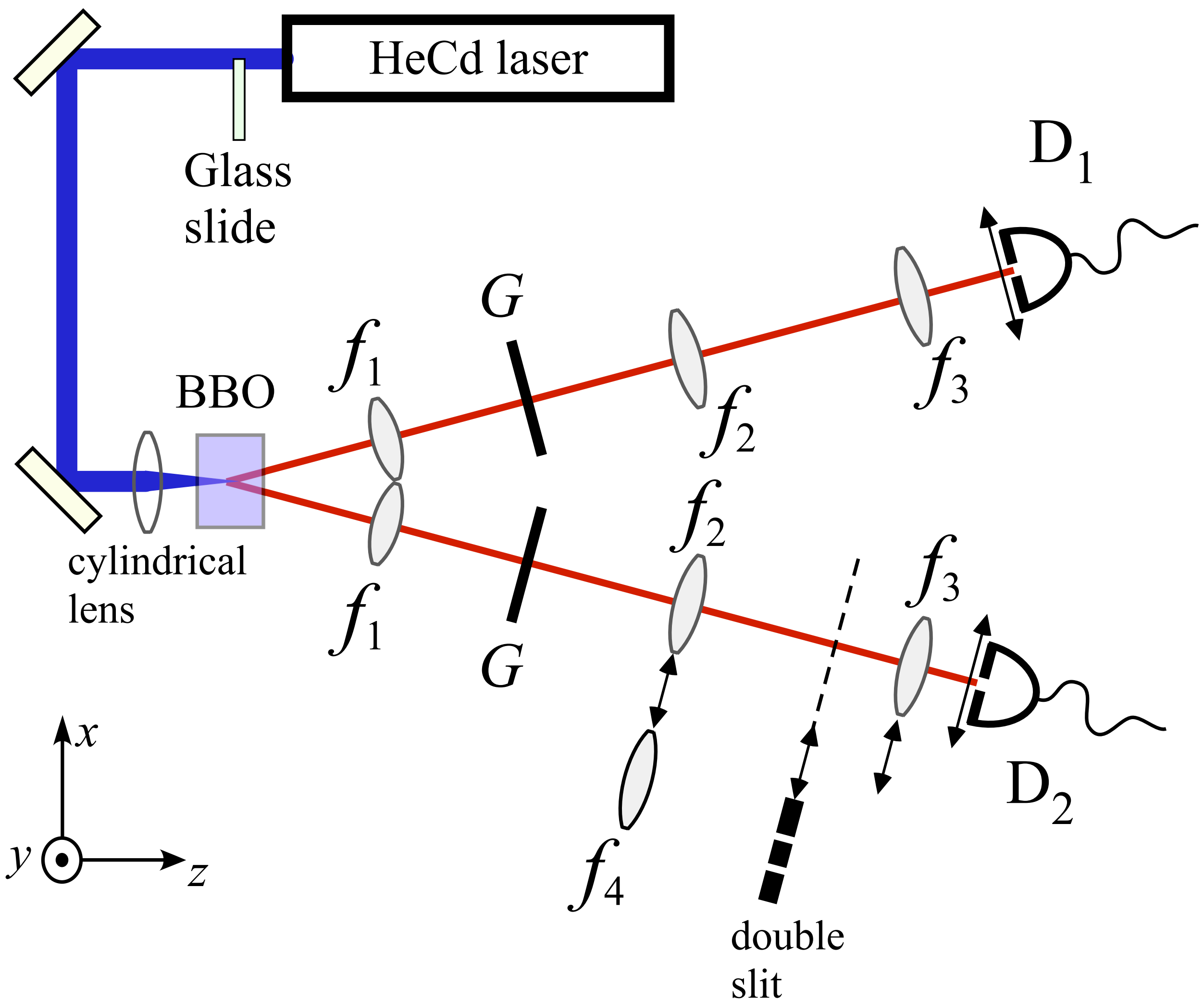
Shifted Young's Fringe Pattern

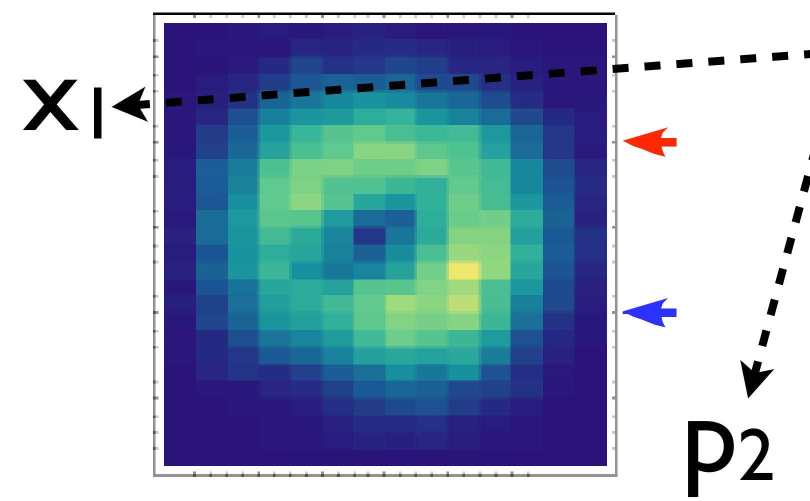


$$I(x,y) = R(x,y)\left[1 + \cos\left(\frac{\alpha(x) + \Delta\phi(y)}{2}\right)\right]$$

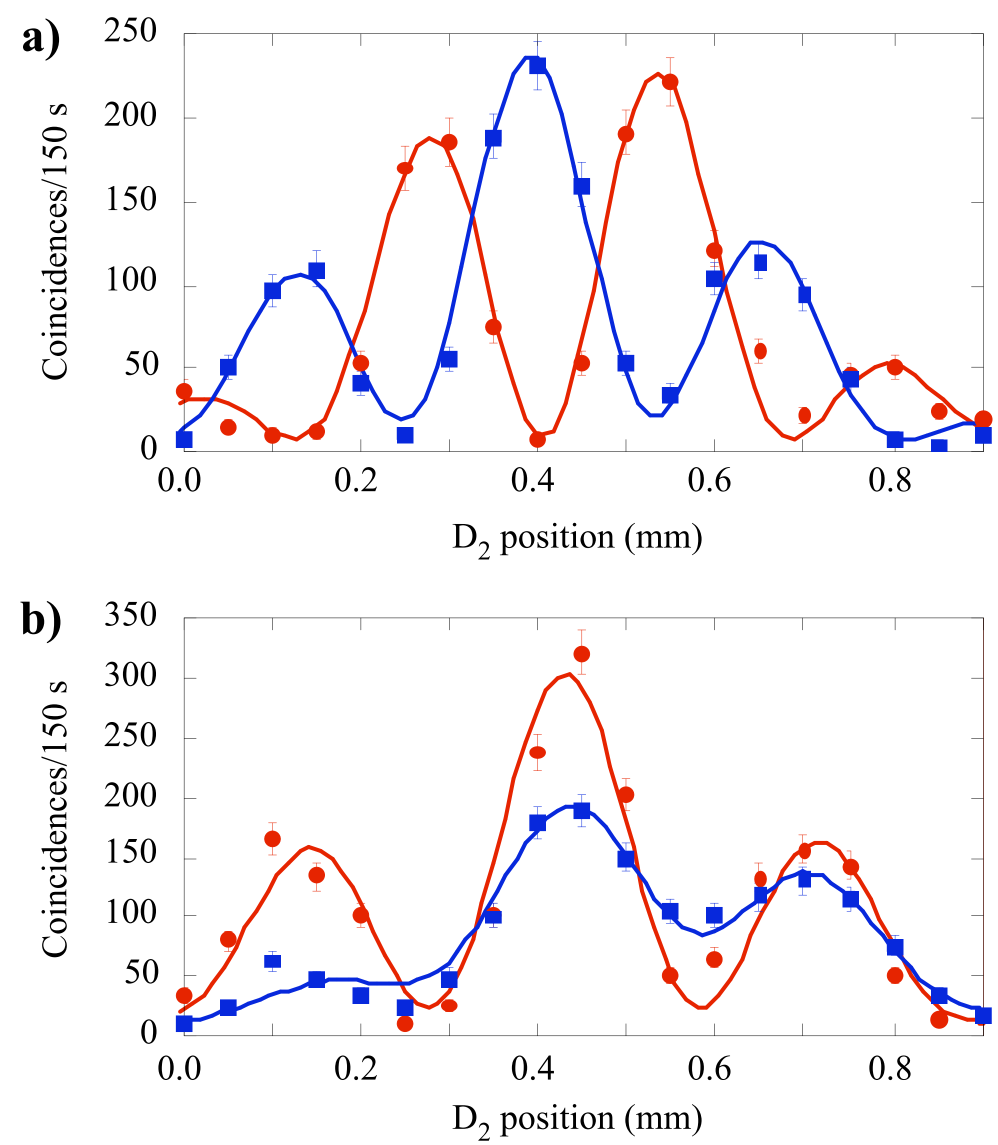
Sztul and Alfano, Optics Letters 31 999 (2006)

“Non-local” optical vortex





Non-local coordinate system



HG pump

Gauss pump

Gomes, Salles, Toscano, Souto Ribeiro, SPW, PRL (2009)

Conclusions

- Spatial Variables of photon pairs - means to investigate aspects of continuous variable entanglement
- Experimental verification of genuine non-Gaussian entanglement. First measurement of a higher-order Shchukin-Vogel criteria?
- Prediction and observation of "non-local" optical vortex. Should be observable in other systems. Uses?
- Experimental investigations led to development of new criteria for entanglement and non-locality

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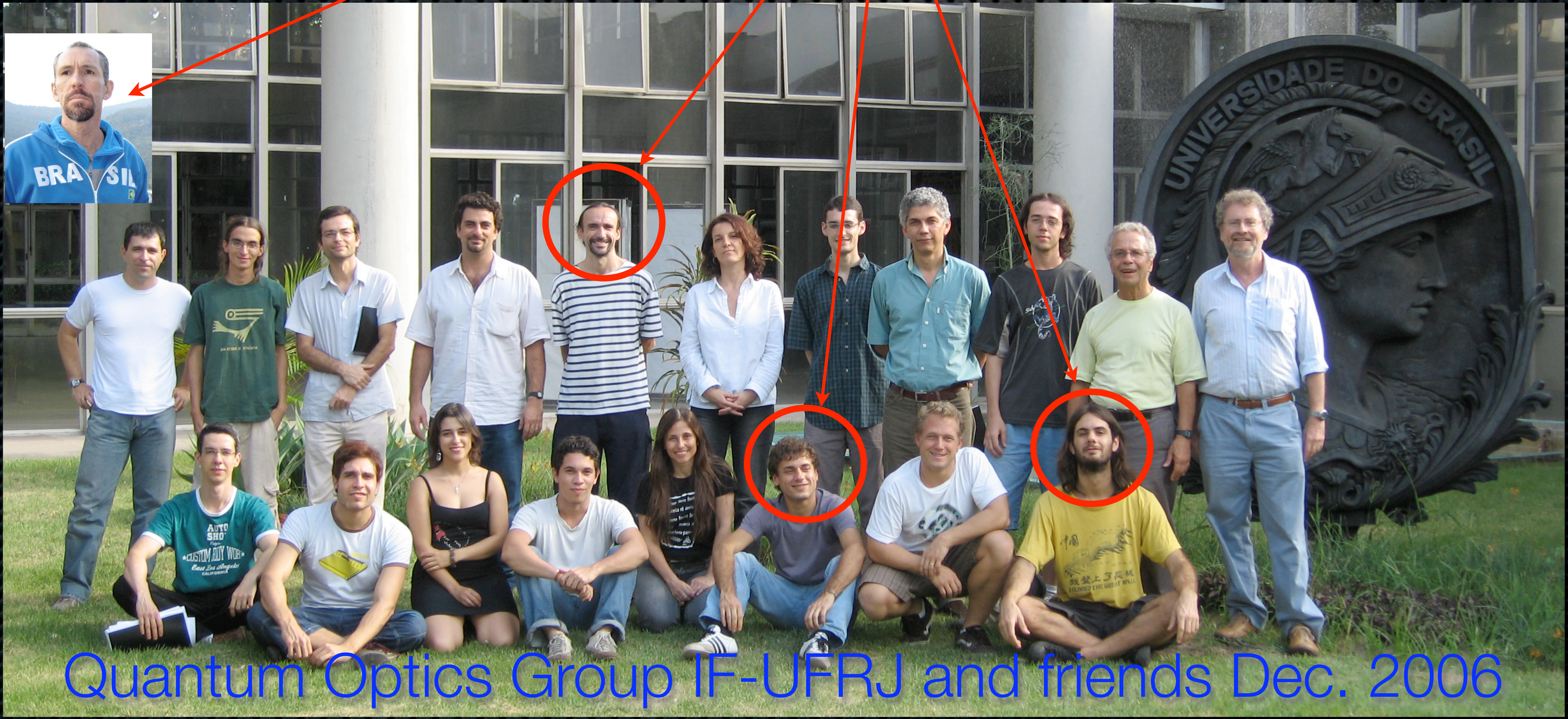
IF - UFRJ

We are always looking for good students and post-docs!

This work:

P. H. Souto Ribeiro (IF-UFRJ)
F. Toscano (IF-UFRJ)
D. S. Tasca (PhD - IF-UFRJ)
A. Salles (IF-UFRJ ->Copenhagen)
Not shown: R. M. Gomes (UFRJ/UFG)



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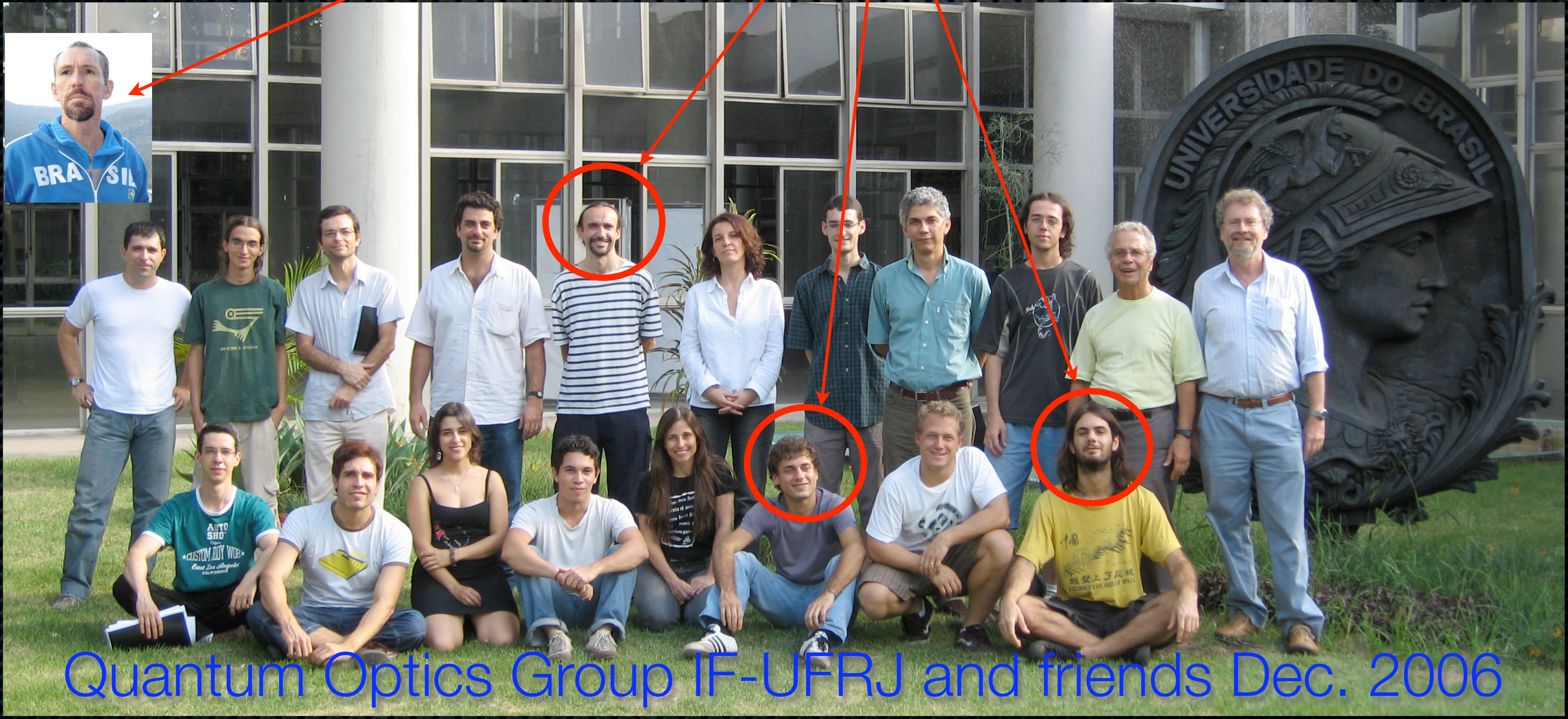


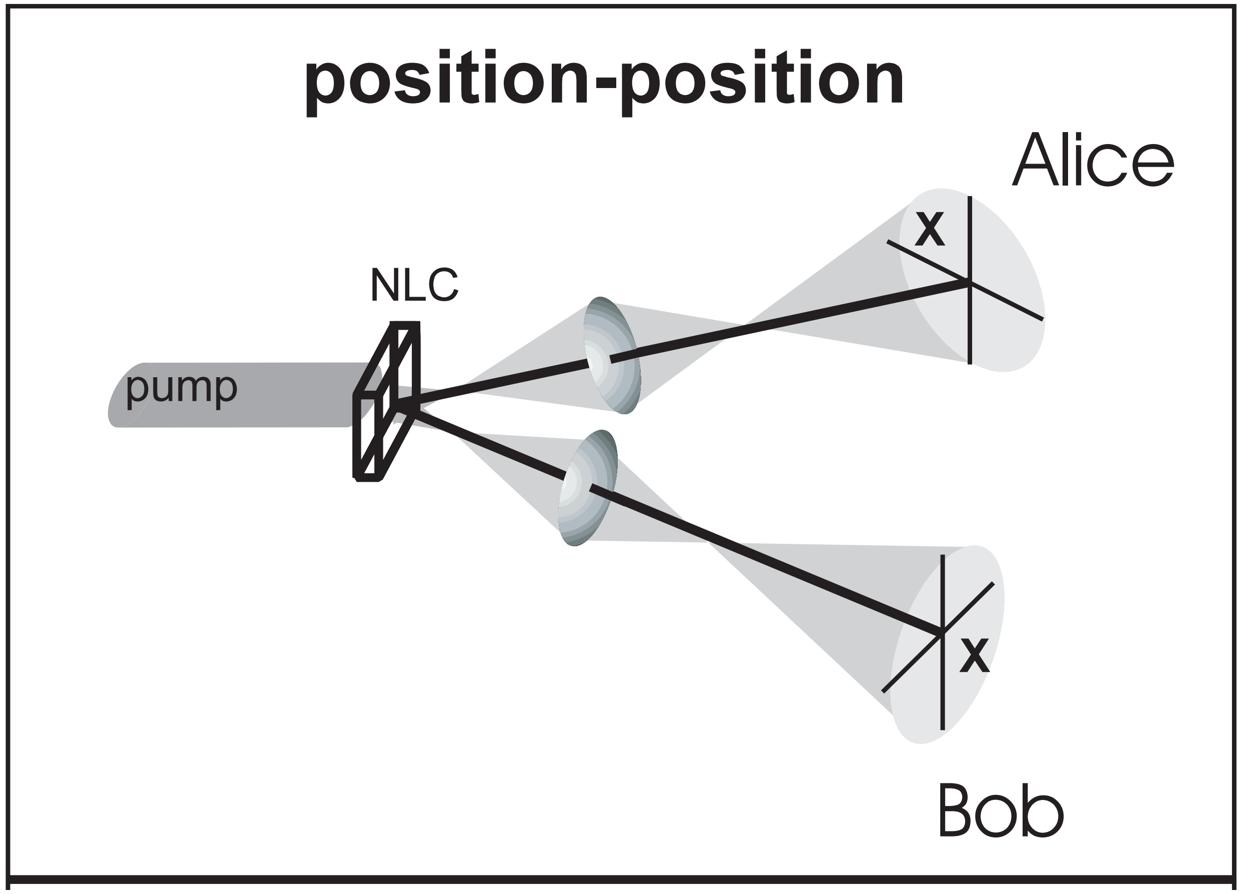
END

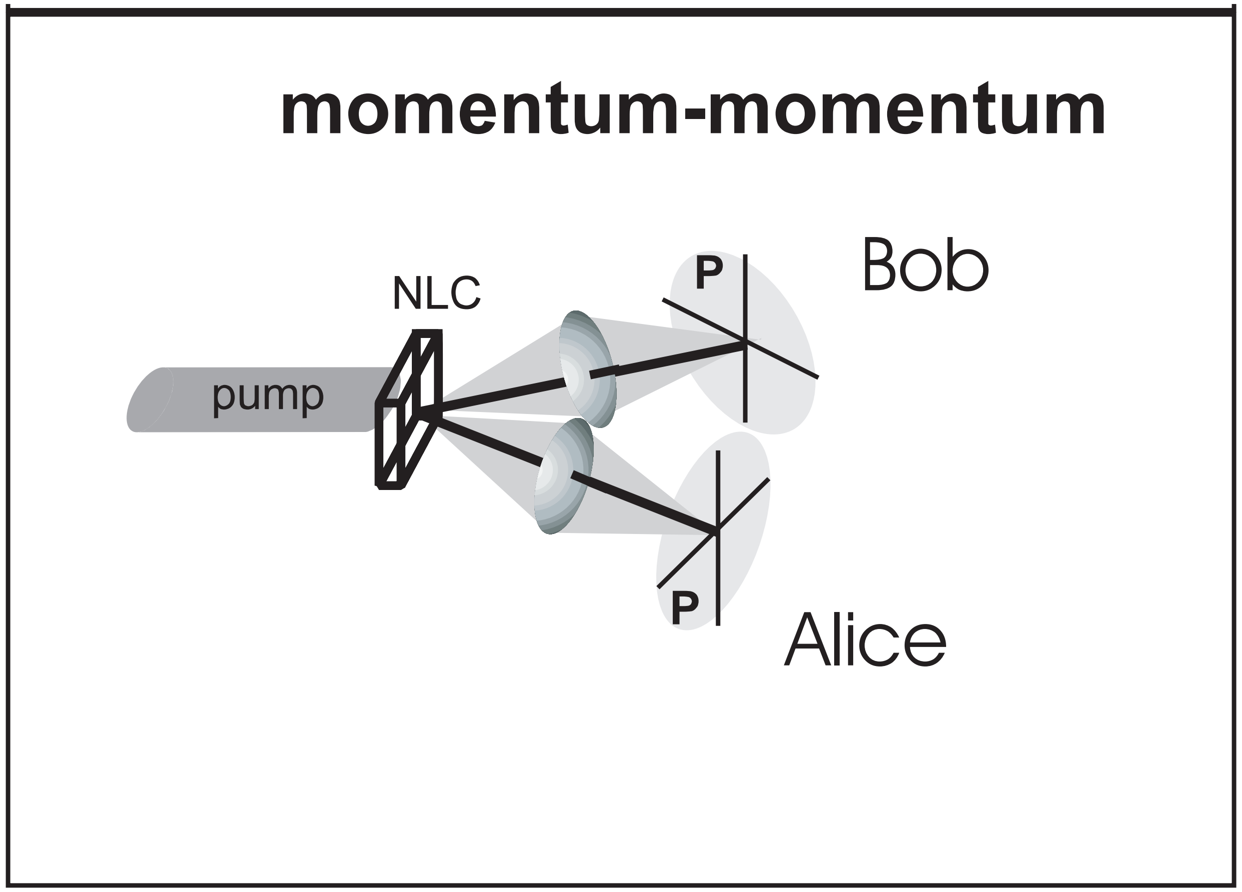
Formal equivalence between usual quantum continuous variables and spatial variables of photons

Position x Momentum p	⟷	Amp. Quadrature x Phase Quadrature p
Photon in Hermite-Gauss mode	⟷	Fock states
Measurement in transverse plane	⟷	Homodyne Detection

Detecting Spatial Entanglement

J. C. Howell, R. S. Bennik, S. J. Bentley and R. W. Boyd PRL **92** (2004).



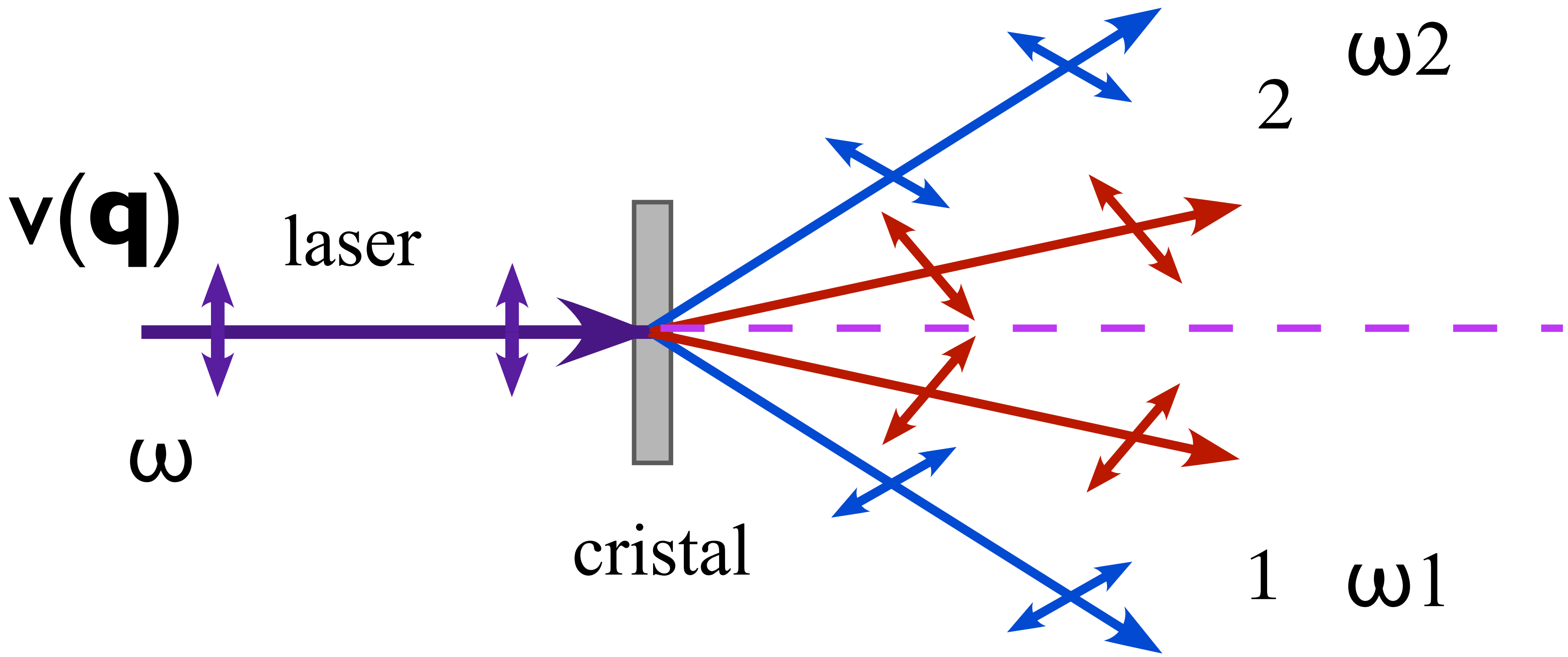


If separable (no entanglement):

$$(\Delta\rho_-)^2(\Delta q_+)^2 \geq 1$$

Mancini, et al PRL 88 (2002).

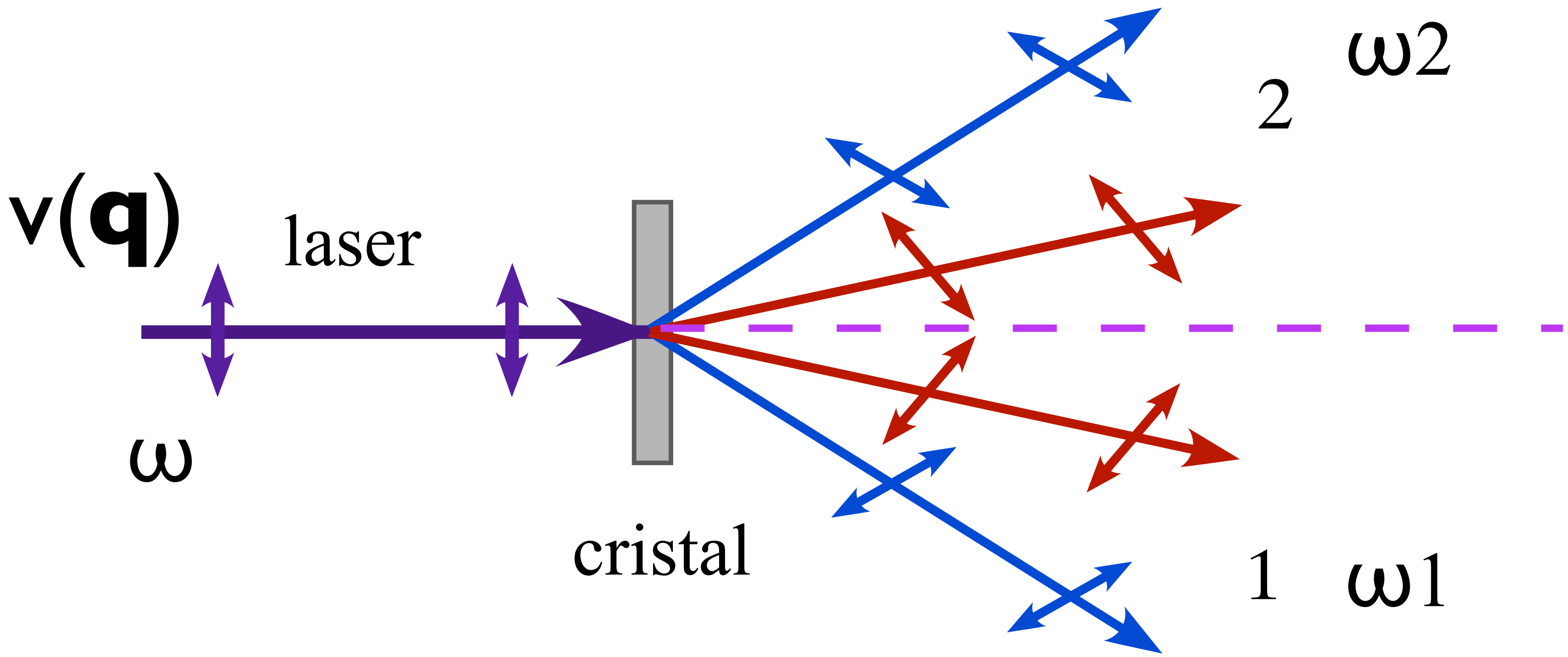
Spontaneous Parametric Down-Conversion (SPDC)



Conservation of Energy: $\omega_1 + \omega_2 = \omega$

Conservation of Transverse Momentum: $\mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}$

Spontaneous Parametric Down-Conversion (SPDC)



Conservation of Energy: $\omega_1 + \omega_2 = \omega$

Conservation of Transverse Momentum: $\mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}$

⇒ Highly correlated quantum states (ENTANGLEMENT)

SPDC: Two-photon state in spatial variables

Monken , Pádua, Souto Ribeiro, PRA 57 3123 (1998).

$$|\psi\rangle = \int \int d\mathbf{q}\, d\mathbf{q}' v(\mathbf{q}+\mathbf{q}')\gamma(\mathbf{q}-\mathbf{q}')|\mathbf{q}\rangle|\mathbf{q}'\rangle$$

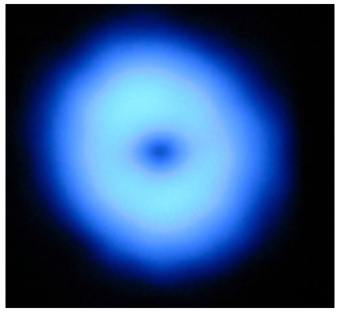
Pump angular spectrum transferred to two-photon state

v(**q**) = angular spectrum of pump field

ϒ(**q**)= phase matching function (crystal)

Entanglement depends on form of functions v and ϒ

First-order LG mode



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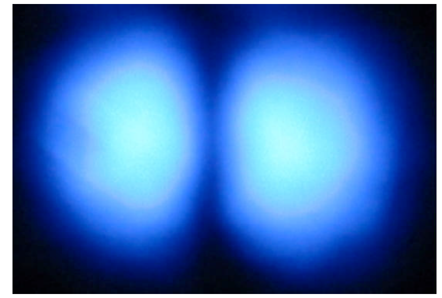

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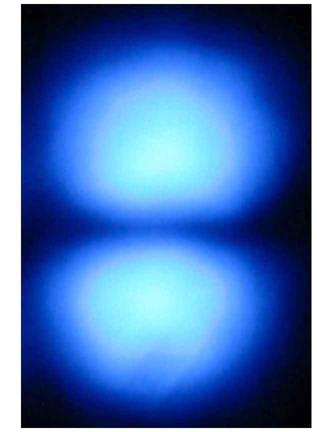

$$\mathrm{U}^{\pm}_{\mathrm{LG}}(x,y) = A\left[u_1(x)u_0(y) \pm i u_0(x) u_1(y)\right]$$

1D Hermite-Gauss functions

$$\mathrm{u}_n(x) = C_n H_n\left(\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{2w(z)^2}\right)\times$$

$$\exp\left\{-i\left[\frac{kx^2}{2R(z)} - \left(n+\frac{1}{2}\right)\varphi(z)\right]\right\}$$

Two-photon "wave function"

$$\psi(\rho_1, \rho_2) = \mathcal{E}(\rho_1 + \rho_2)\Gamma(\rho_1 - \rho_2)$$

$$\psi(\rho_1, \rho_2) = \mathcal{U}_{10}(\rho_1 + \rho_2, \sigma)\mathcal{U}_{00}(\rho_1 - \rho_2, \delta)$$

LG mode in "non-local" coordinates

$$\psi(\rho_1, q_2) \propto [u_1(\rho_{x1})u_0(q_{x2}) + iu_0(\rho_{x1})u_1(q_{x2})]\, u_0(\rho_{y1})u_0(q_{y2})$$

Appears only in correlations: "non-local" optical vortex