E LÁ SE PASSARAM MAIS DE 20 ANOS DE MECÂNICA ESTATÍSTICA NÃO EXTENSIVA…

O QUE HÁ DE NOVO?

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Niteroi, Maio 2009

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, 1981), page 35

> *In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92)* [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space [$\cdots$]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. [$\cdots$]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

POSTULATE FOR THE ENTROPIC FUNCTIONAL

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG **entropy** **(*q* = 1)**	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* **(*q* real)**	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production

Composable

Topsoe-factorizable

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

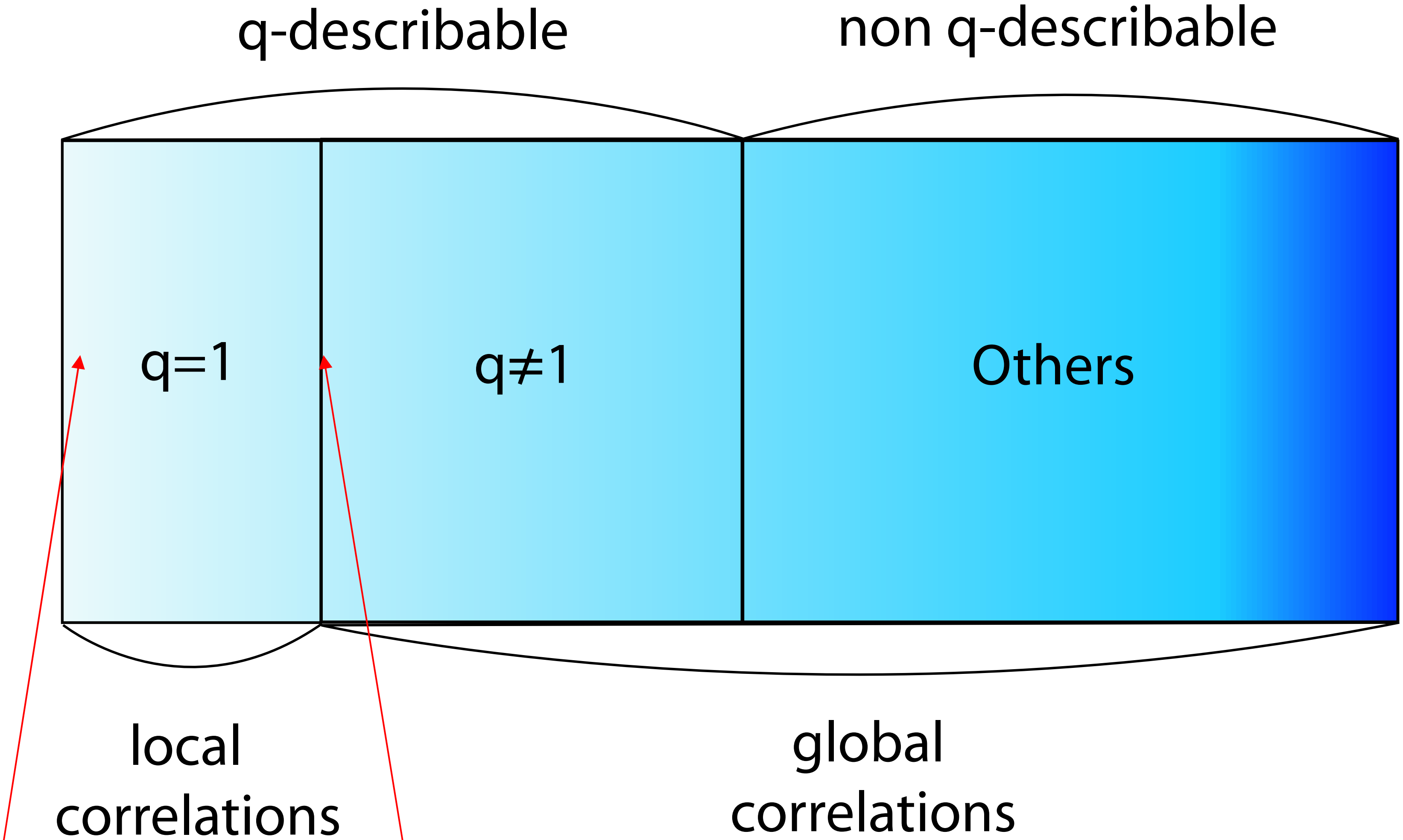
DEFINITION (q – *logarithm*):

$$\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0)$$

$$\ln_1 x = \ln x$$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$



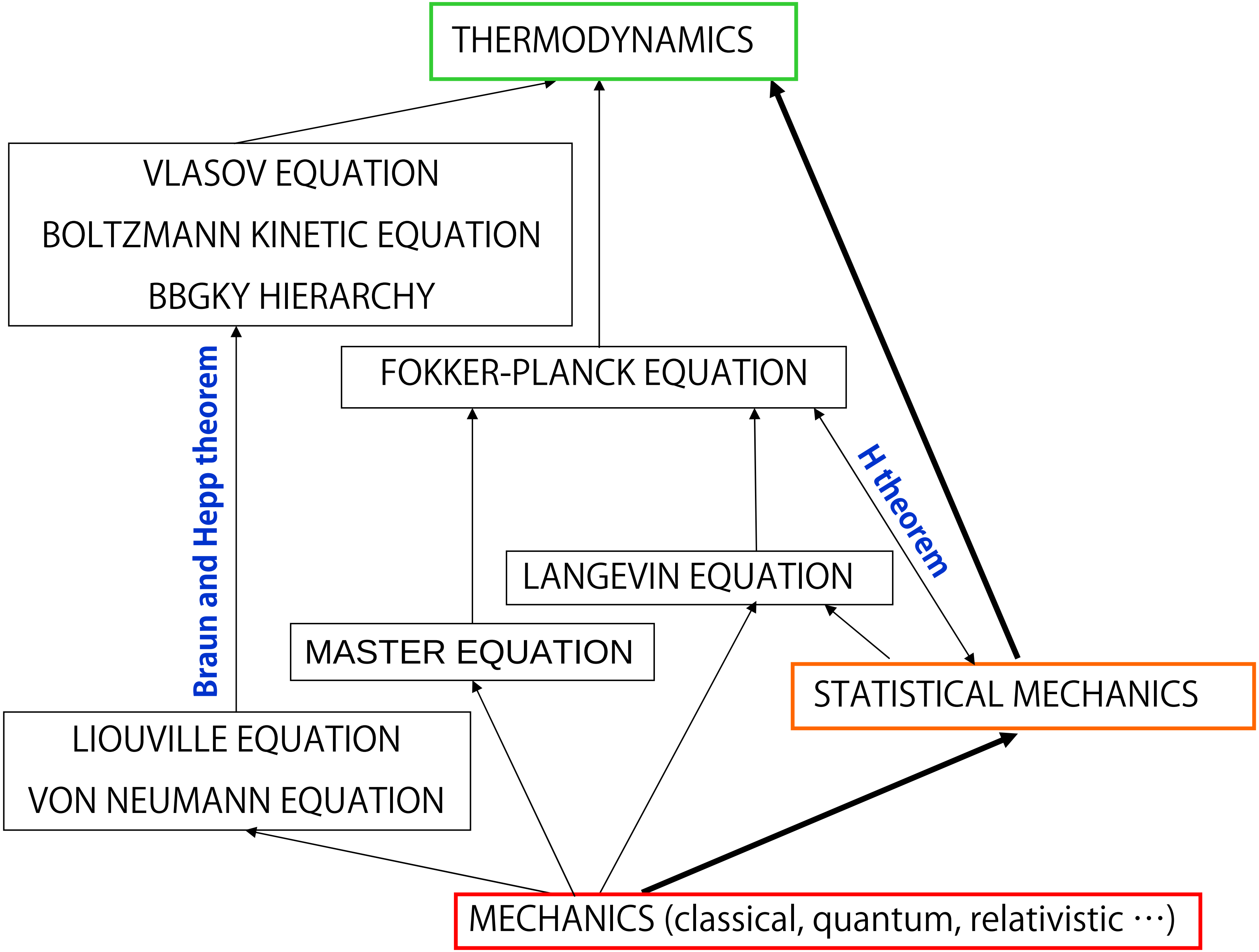
IDEAL GAS

CRITICAL PHENOMENA

$$q = \frac{1+\delta}{2}$$ [A. Robledo, Mol Phys 103 (2005) 3025]

$$q = \frac{\sqrt{9+c^2}-3}{c}$$ [F. Caruso and C. T., Phys Rev E 78 (2008) 021101]

C.T., M. Gell-Mann and Y. Sato
Europhysics News **36** (6), 186
(European Physical Soc., 2005)


THERMODYNAMICS
VLASOV EQUATION
BOLTZMANN KINETIC EQUATION
BBGKY HIERARCHY
FOKKER-PLANCK EQUATION
Braun and Hepp theorem
H theorem
LANGEVIN EQUATION
MASTER EQUATION
STATISTICAL MECHANICS
LIOUVILLE EQUATION
VON NEUMANN EQUATION
MECHANICS (classical, quantum, relativistic ···)

GROUNDING STAT. MECH.: Entropy extremization

Extremization of S_q with appropriate constraints yields

$$p_q(x) \propto [1-(1-q)\beta\, x]^{\frac{1}{1-q}} \equiv e_q^{-\beta\, x}$$ (q-generalized Boltzmann weight)

[inverse function of $\ln_q x$]

If $\langle x \rangle = 0,$ extremization of S_q yields instead

$$p_q(x) \propto [1-(1-q)\beta\, x^2]^{\frac{1}{1-q}} \equiv e_q^{-\beta\, x^2}$$ (q-generalized Gaussian)

EXTENSIVITY OF THE NONADDITIVE ENTROPY S_q

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

Hence, S_{BG} and $S_q^{Renyi}\,(\forall q)$ are additive, and S_q $(\forall q\neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma\equiv A_1+A_2+\ldots+A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N. An entropy is extensive if

$$0<\lim_{N\to\infty}\frac{S(N)}{N}<\infty\;,\; i.e.,\;\; S(N)\propto N\;\;(N\to\infty)$$

CONSEQUENTLY:

The additive entropies S_{BG} and S_q^{Renyi} are extensive if and only if the N subsystems are (strictly or asymptotically) independent; otherwise, S_{BG} and S_q^{Renyi} are nonextensive.

The nonadditive entropy S_q $(q\neq 1)$ is extensive for special values of q if the subsystems are specially (globally) correlated.

HYBRID PASCAL - LEIBNITZ TRIANGLE

(N=0) $1\times\frac{1}{1}$

(N=1) $1\times\frac{1}{2}$ $1\times\frac{1}{2}$

(N=2) $1\times\frac{1}{3}$ $2\times\frac{1}{6}$ $1\times\frac{1}{3}$

(N=3) $1\times\frac{1}{4}$ $3\times\frac{1}{12}$ $3\times\frac{1}{12}$ $1\times\frac{1}{4}$

(N=4) $1\times\frac{1}{5}$ $4\times\frac{1}{20}$ $6\times\frac{1}{30}$ $4\times\frac{1}{20}$ $1\times\frac{1}{5}$

(N=5) $1\times\frac{1}{6}$ $5\times\frac{1}{30}$ $10\times\frac{1}{60}$ $10\times\frac{1}{60}$ $5\times\frac{1}{30}$ $1\times\frac{1}{6}$

$$\sum_{n=0}^{N}\binom{N}{n} r_{N,n} = 1 \quad (\forall\ N)$$

Blaise **Pascal** (1623-1662)
Gottfried Wilhelm **Leibnitz** (1646-1716)
Daniel **Bernoulli** (1700-1782)

$(N=2)$

B \ A	1	2	
1	$p^2+\kappa$	$p(1-p)-\kappa$	p
2	$p(1-p)-\kappa$	$(1-p)^2+\kappa$	1- p
	p	1- p	1

EQUIVALENTLY:

$(N=0)$ $\quad 1\times 1$

$(N=1)$ $\quad 1\times p \qquad 1\times(1-p)$

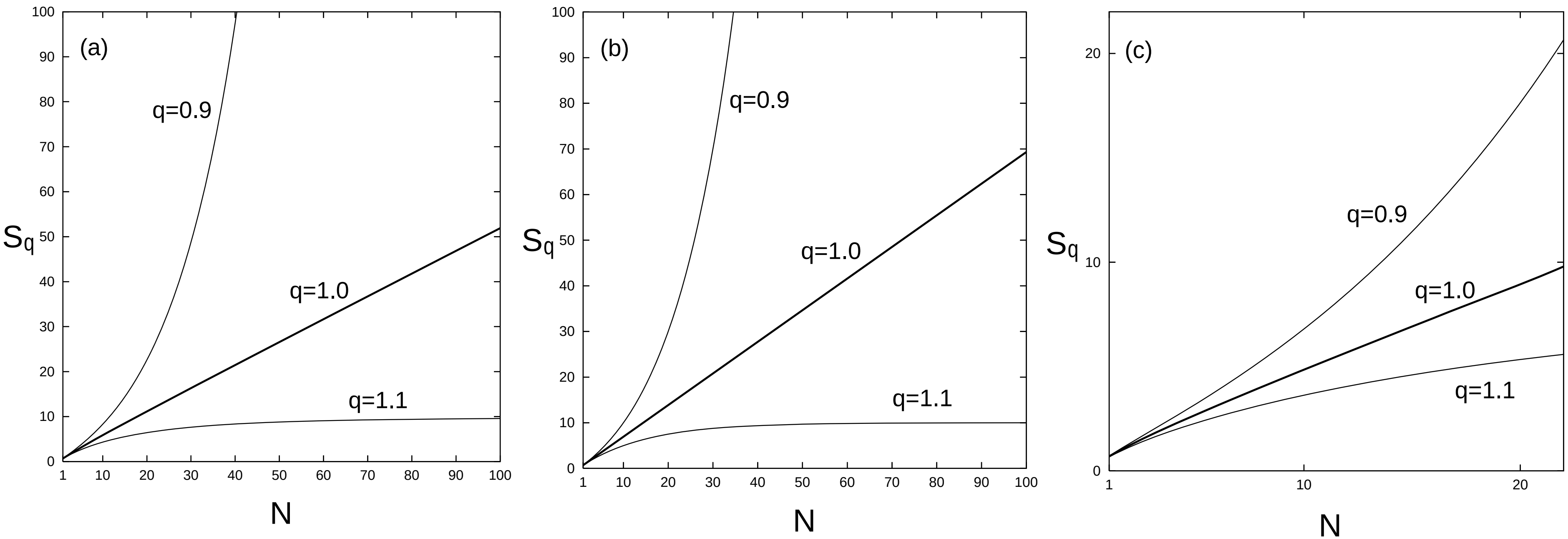
$(N=2)$ $\quad 1\times[p^2+\kappa] \qquad 2\times[p(1-p)-\kappa] \qquad 1\times[(1-p)^2+\kappa]$

$q = 1$ SYSTEMS

i.e., such that $S_1(N) \propto N \quad (N \to \infty)$

I don't believe that atoms exist!

Ernst Mach (January 1897, Vienna)



Leibnitz triangle

$$\left(p_{N,0} = \frac{1}{N+1} \right)$$

N independent coins

$$\left(\begin{array}{l} p_{N,0} = p^{N} \\ \\ \text{with } p = 1/2 \end{array} \right)$$

Stretched exponential

$$\left(\begin{array}{l} p_{N,0} = p^{N^{\alpha}} \\ \\ \text{with } p = \alpha = 1/2 \end{array} \right)$$

(All three examples **strictly** satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

Asymptotically scale-invariant (d=2)

$$
\begin{array}{lccccccccc}
(N=0) & & & & & 1 & & & & \\
(N=1) & & & & 1/2 & & 1/2 & & & \\
(N=2) & & & 1/3 & & 1/6 & & 1/3 & & \\
(N=3) & & 3/8 & & 5/48 & & 5/48 & & 0 & \\
(N=4) & 2/5 & & 3/40 & & 1/20 & & 0 & & 0
\end{array}
$$

$\xleftrightarrow{\quad d+1 \quad}$

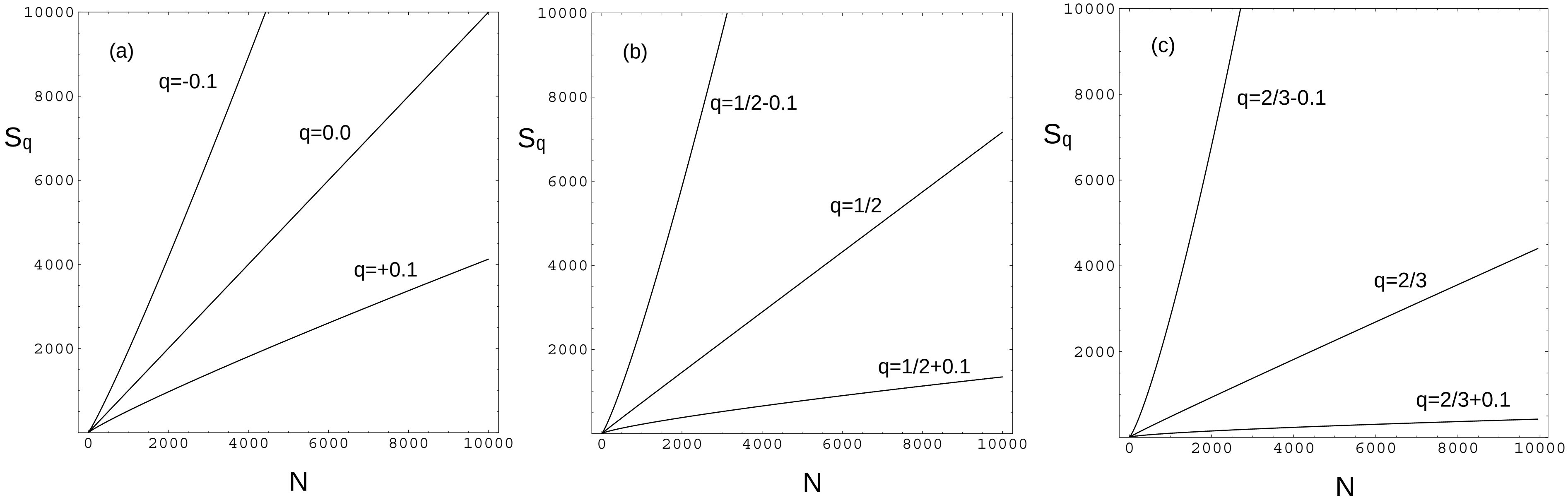
(It **asymptotically** satisfies the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

$q \neq 1$ SYSTEMS

i.e., such that $S_q(N) \propto N \quad (N \to \infty)$

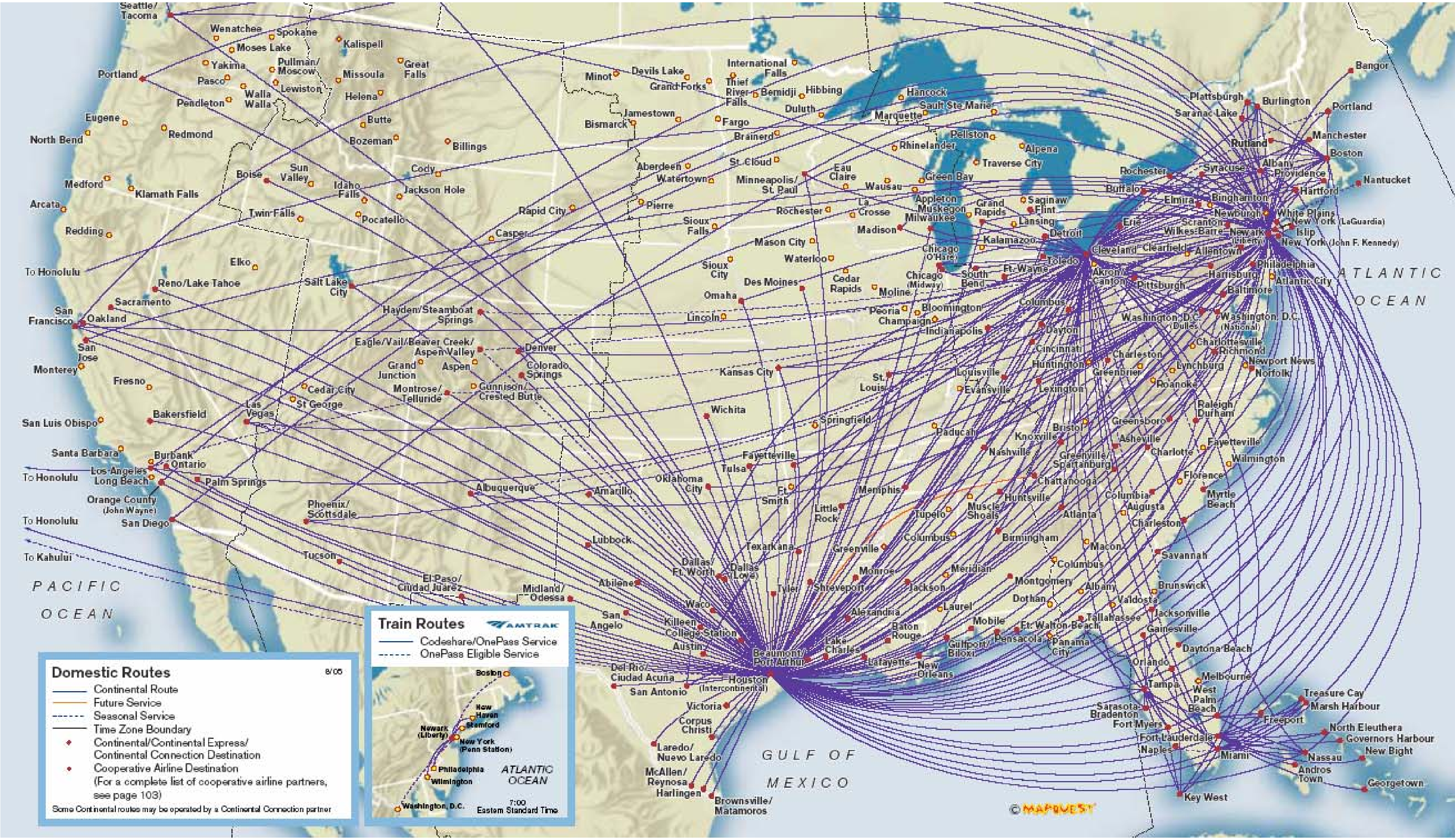
$(d=1)$ $(d=2)$ $(d=3)$



$$q = 1 - \frac{1}{d}$$

(All three examples **asymptotically** satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)



Continental Airlines

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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(Received 16 March 2008; revised manuscript received 16 May 2008; published 5 August 2008)

The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z\right]$$

$|\gamma| = 1 \quad \rightarrow \quad$ *Ising ferromagnet*

$0 < |\gamma| < 1 \quad \rightarrow \quad$ *anisotropic XY ferromagnet*

$\gamma = 0 \quad \rightarrow \quad$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

$\rho_N \equiv$ ground state $(T=0)$ of the N-system

(assuming $\lambda^{xy} = +0$)

$\Rightarrow \rho_N^2 = \rho_N \Rightarrow Tr\,\rho_N^2 = 1$

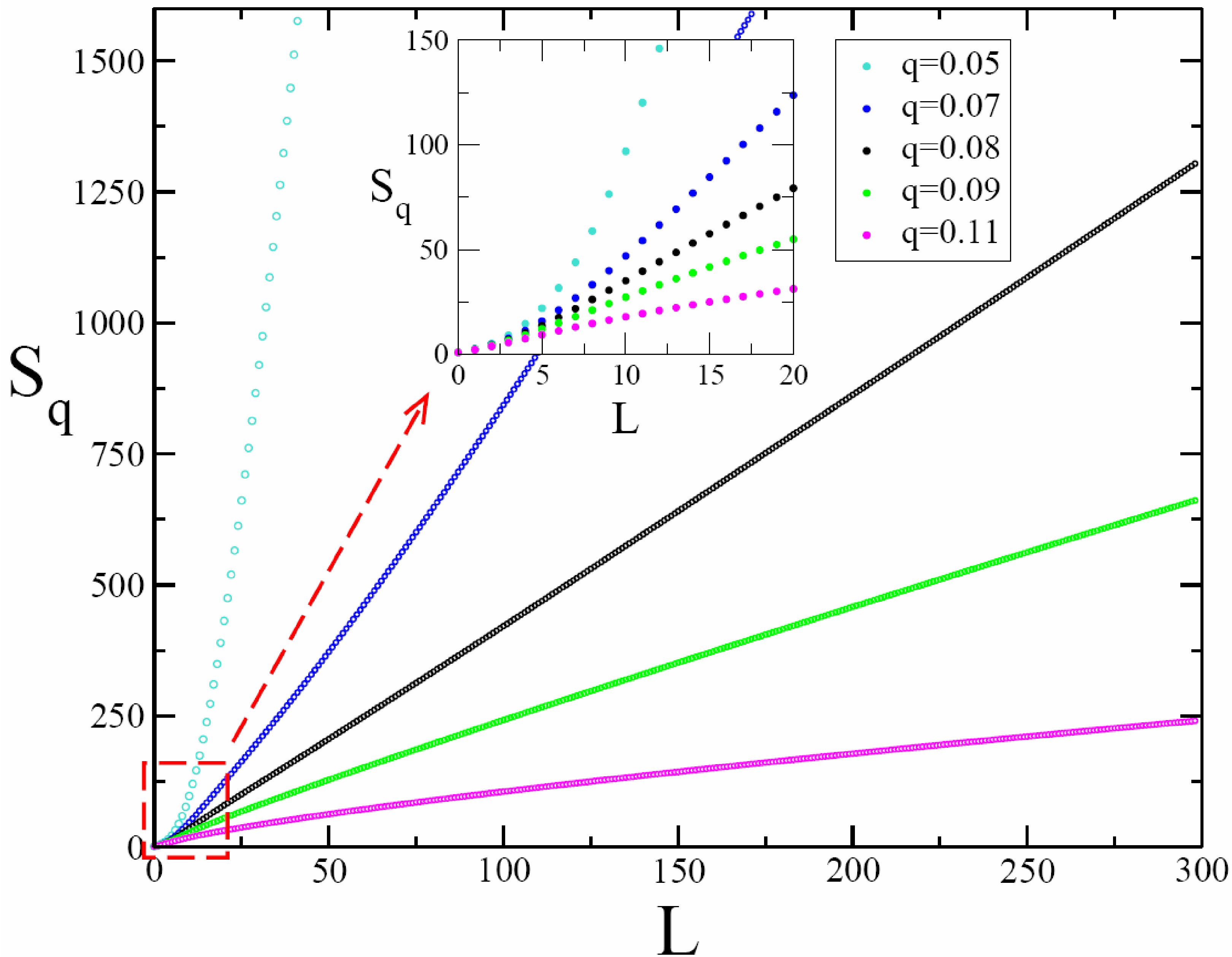
$\Rightarrow \rho_N$ is a pure state

$\Rightarrow S_q(N) = 0 \quad (\forall q,\ \forall N)$

Whereas $\rho_L \equiv Tr_{N-L}\ \rho_N$ satisfies $Tr\,\rho_L^2 < 1$

$\Rightarrow \rho_L$ is a mixed state

$\Rightarrow S_q(N, L) > 0$



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

*in P. Calabrese and J. Cardy, JSTAT P*06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

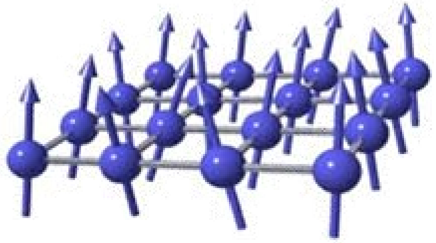
Ising and anisotropic XY ferromagnets $\Rightarrow\ c = \frac{1}{2}\ \Rightarrow\ q_{ent} = \sqrt{37}-6\ \approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow\ c = 1\ \Rightarrow\ q_{ent} = \sqrt{10}-3\ \approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

2-D quantum systems at T=0

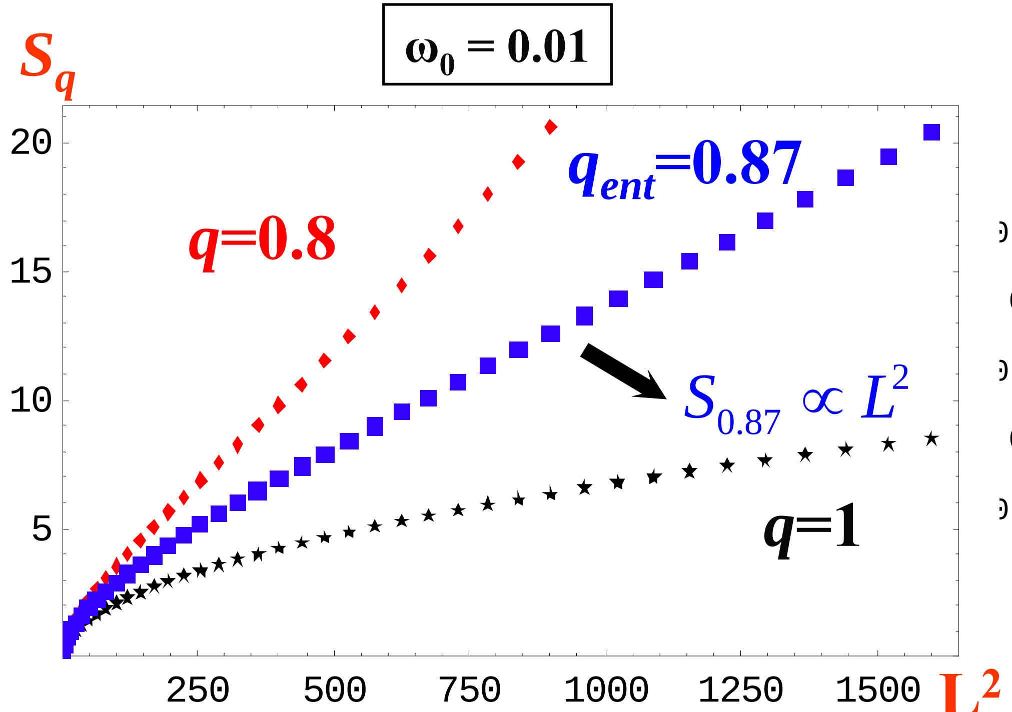


Bosonic two-dimensional system of infinite coupled harmonic oscillators at T=0

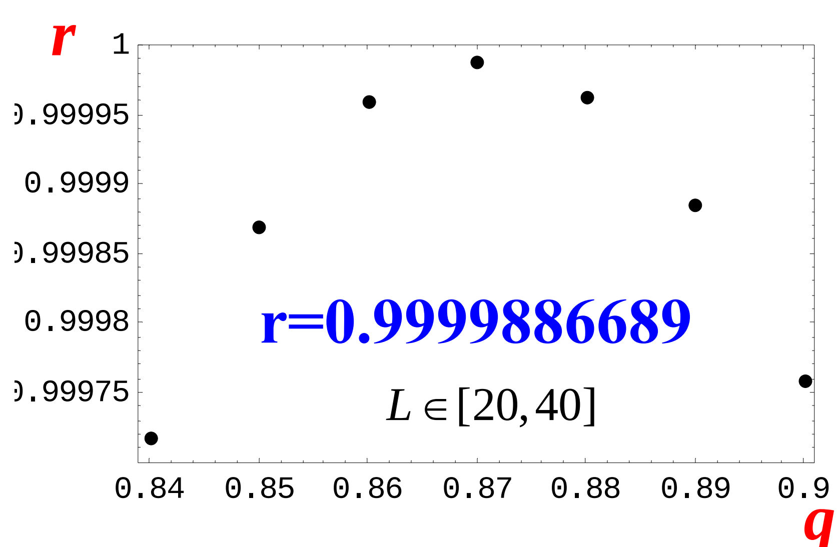
$$H = \frac{1}{2}\sum_{x,y}[\Pi_{x,y}^2 + \omega_0^2\Phi_{x,y}^2 + (\Phi_{x,y} - \Phi_{x+1,y})^2 + (\Phi_{x,y} - \Phi_{x,y+1})^2]$$

self-frequency **coordinate** **momentum**

(the masses and coupling strengths are set to unity)



Extensivity for q_{ent}=0.87



F. Caruso and C. T., Phys Rev E **78,** 021101 (2008)

Summarizing, for a wide class of quantum problems, we know that

$$
\begin{aligned}
S_{BG}(N) &\propto \ln L \propto \ln N \neq N && \text{for } d=1 \text{ quantum chains} \\
&\propto L \propto \sqrt{N} \neq N && \text{for } d=2 \text{ bosonic systems} \\
&\propto L^2 \propto N^{2/3} \neq N && \text{for } d=3 \text{ black hole} \\
&\propto L^{d-1} \propto N^{(d-1)/d} \neq N && \text{for } d\text{-dimensional bosonic systems} \\
& && (d>1;\ \text{area law}) \\
&\propto \frac{L^{d-1}-1}{d-1} \equiv \ln_{2-d} L \neq L^d \propto N && (d \geq 1) \qquad \text{(NONEXTENSIVE!)}
\end{aligned}
$$

For the same class of quantum problems, we expect

$$
S_{q_{ent}}(N) \propto L^d \propto N \qquad (d \geq 1;\ q_{ent} \neq 1) \qquad \text{(EXTENSIVE!)}
$$

(which we have illustrated for $d=1,2$)

F. Caruso and C. T., Phys Rev E **78,** 021101 (2008)

When entropy does not seem extensive

Earlier speculations about the entropy of black holes has prompted an ingenious calculation suggesting that entropy may (in special circumstances) be the same inside and outside an arbitrary boundary.

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. That, of course, is why the entropy of some substance will be quoted as so much per gram, or mole. If you then take two grams, or two moles, of the same material under the same conditions, the entropy will be twice as much. And there should be no confusion about the units; the simple Carnot definition of a change of entropy in a reversible process is the heat transfer divided by the absolute temperature, so that the units of entropy are simply those of energy divided by temperature, joules per degree (kelvin) in the SI system. The definitions of the Gibbs and Helmholtz free energies would be dimensionally discordant for that reason were it not that entropy (S) always turns up multiplied by temperature T. So much will readily be agreed.

Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? By the number of ways in which the constituents of some material (the atoms and molecules) can be rearranged without changing its properties and without energetic consequences. But now there comes a snag.

Like any extensive property, the combined entropy of two separate chunks of material should be the *sum* of the two entropies, but the number of rearrangements of the combined system must be the *product* of the numbers of ways in which the two parts separately can be rearranged. How to reconcile that with extensivity? By supposing entropy is proportional not to the number of rearrangements (technically called 'complexions'), but with the logarithm thereof. And because entropy decreases as disorder increases, the constant of proportionality must be a negative (real) number.

From that it follows that $S = S_0 - K\log N$, where K is a positive constant with the dimensions of entropy, N is a number (without dimensions) measuring disorder and S_0 is an arbitrary constant entropy. All that is simply a precis of the standard introductory chapter in statistical mechanics textbooks, most of which go on to show how to calculate the properties of assemblages of, say, diatomic molecules from a knowledge of their individual behaviour. Because the number of complexions of a particular state of an assemblage is invariably a function of the number (n) of molecules it contains, usually in the form of $n!$, because n is usually large and because $\log(n!)$ can then be approximated by $n\log n$, the extensive property of entropy then follows simply from the appearance of the leading factor n: entropy is proportional to the number of molecules.

That is what the textbooks say. It also makes sense of what is known of the thermodynamics of the real world. In a sample of a diatomic gas, for example, there are vibrations (one) and rotations (two) as well as three rectilinear degrees of freedom. But the problem is to tell how the energy available is distributed among the different degrees of freedom. The arithmetic simplifies marvellously because (in this case) each molecule and each of its degrees of freedom is independent. The best measure of disorder works out at $N = Z^n$, where n is the number of molecules, and where Z, which must be a [...]

A bit of quantum mechanics goes into the argument as well, notably the notion of the density matrix — an artificially constructed operator (on quantum states) that is well suited to the discussion of systems in which one part (say the black hole) is singled out for attention while the remainder (the Universe outside it) is dealt with in less detail, perhaps because some averaging process is appropriate, or because the whole problem may not be calculable at all. (In Dirac's notation, the density matrix corresponding to some state of the whole Universe would be represented as $|1\rangle\langle 1|$, where "1" is simply the name for a particular state of the Universe.) What matters, where entropy is concerned, is that the density matrix, like all matrices, has eigenvalues from which the entropy can be calculated.

So imagine that the Universe is partitioned into two parts by means of a closed boundary of some kind and filled with a [...]

dealt with explicitly, as other entropy calculations are made. And that could be exceedingly important.

John Maddox

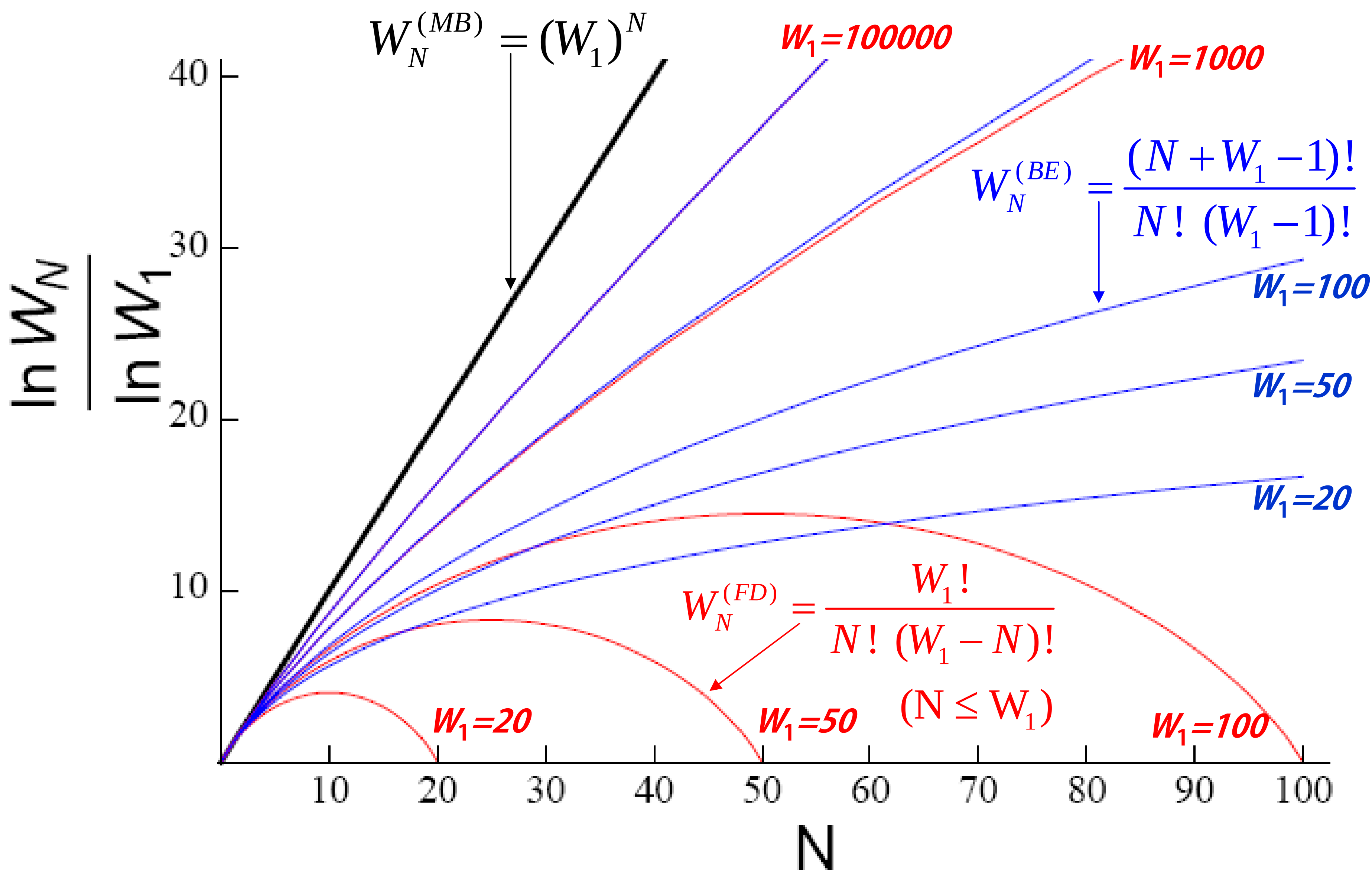
Jacob D. Bekenstein
Stephen W. Hawking
Gerard 't Hooft
Leonard Susskind
Stephen Lloyd Juan M. Maldacena …

When entropy does not seem extensive
John Maddox, Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

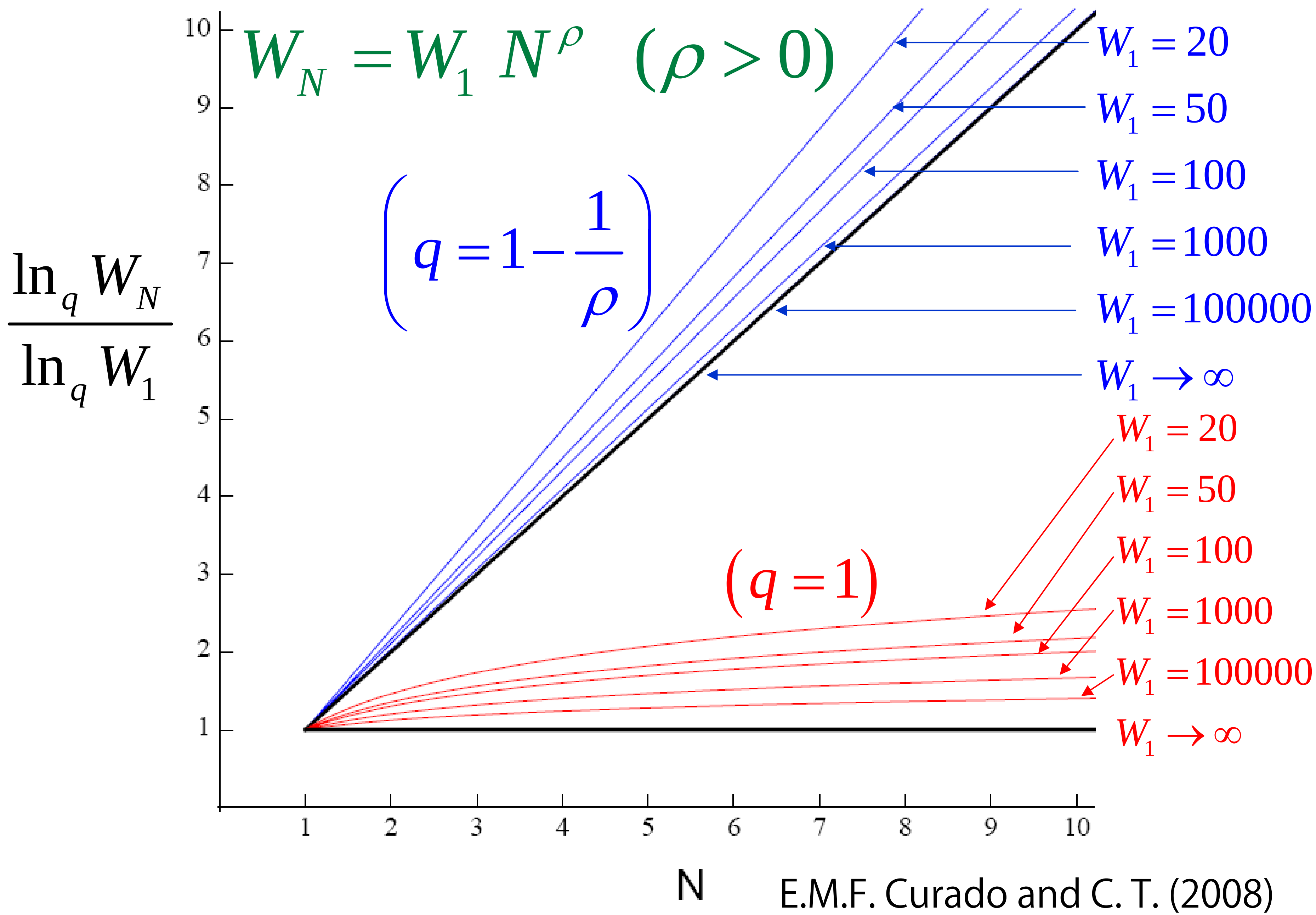
SYSTEMS	ENTROPY S_{BG} (additive)	ENTROPY S_q (q<1) (nonadditive)
Short-range interactions, weakly entangled blocks, etc	EXTENSIVE	NONEXTENSIVE
Long-range interactions (QSS), strongly entangled blocks, etc	NONEXTENSIVE	EXTENSIVE

MICROCANONICAL ENSEMBLE



E.M.F. Curado and C. T. (2008)

MICROCANONICAL ENSEMBLE



E.M.F. Curado and C. T. (2008)

q- GENERALIZATION OF THE CENTRAL LIMIT THEOREM

ONE OF MANY CONNECTIONS OF THE CENTRAL LIMIT THEOREM WITH BOLTZMANN-GIBBS STATISTICAL MECHANICS

Optimization of

$$S = -\,k\int dx\ p(x)\ \ln[p(x)]$$

with

$$\int dx\ p(x) = 1$$

and

$$\langle E(x)\rangle \equiv \int dx\ p(x)\ E(x) = constant$$

yields

$$p(x) = \frac{e^{-\beta E(x)}}{\int dy\ e^{-\beta E(y)}}$$

(Boltzmann-Gibbs distribution for thermal equilibrium)

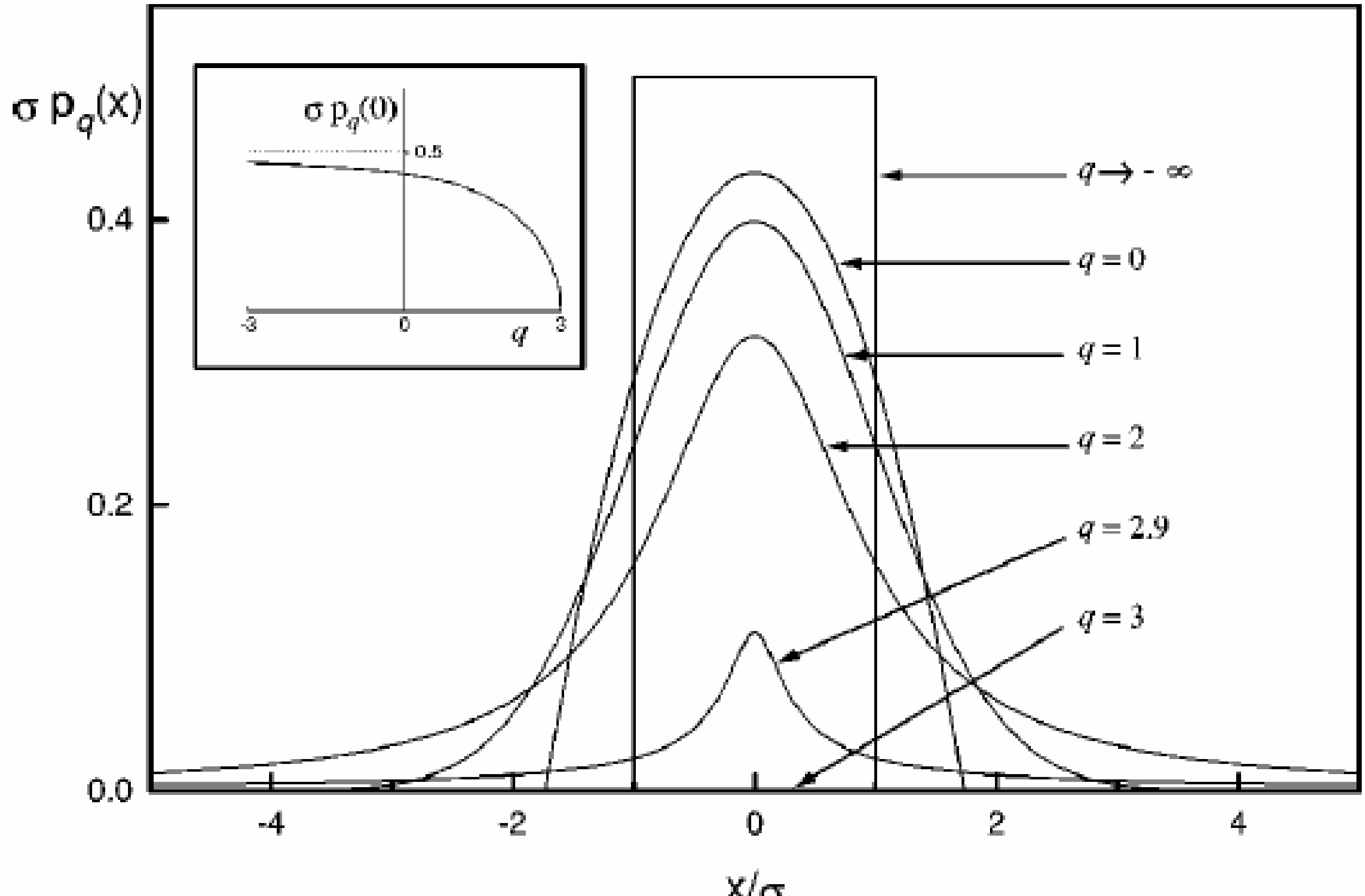
Example: $\langle x\rangle = 0$ *and* $\langle x^2\rangle = constant$ *yields*

$$p(x) = \frac{e^{-\beta x^2}}{\int dy\ e^{-\beta y^2}}$$

(Gaussian distribution)

q-GAUSSIANS:

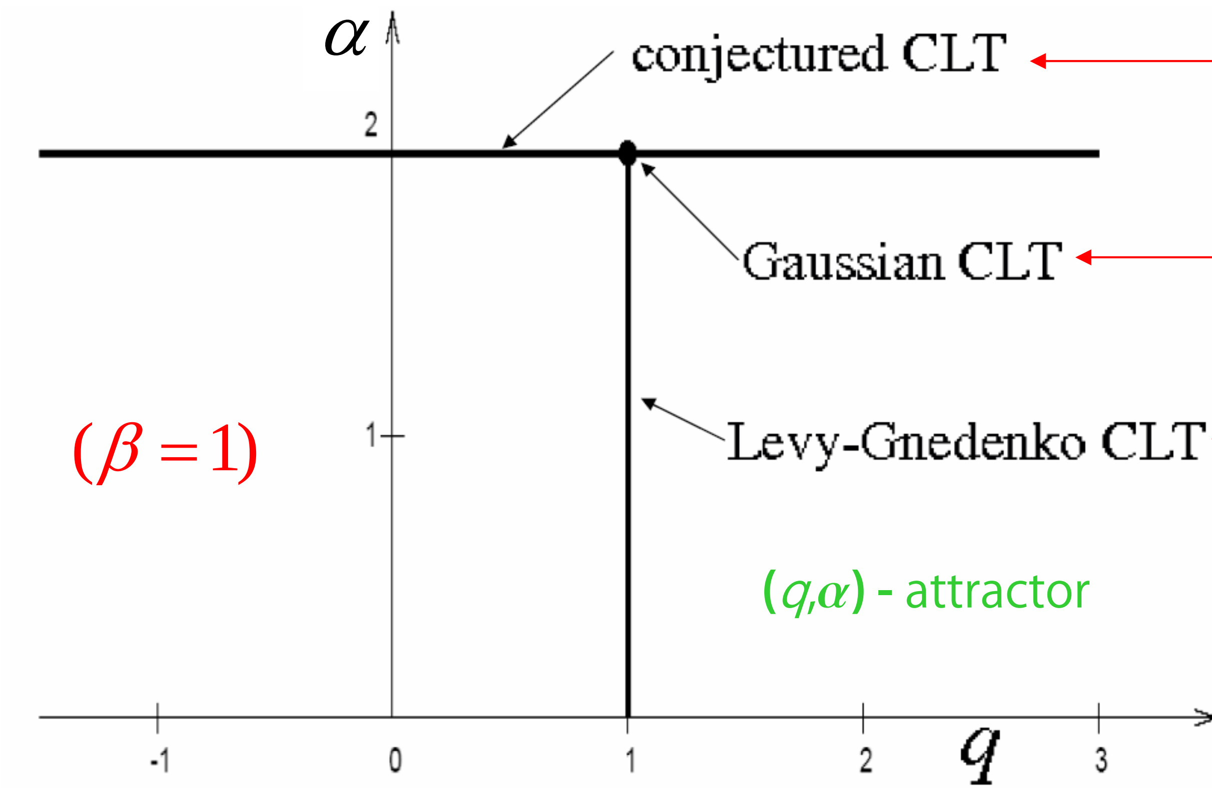
$$p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}} \qquad (q<3)$$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

LOOKING FOR A q-GENERALIZED CENTRAL LIMIT THEOREM:

$$\frac{\partial^{\beta} p(x,t)}{\partial t^{\beta}} = D \frac{\partial^{\alpha} [p(x,t)]^{2-q}}{\partial |x|^{\alpha}} \quad (0 < \alpha \le 2;\ q < 3;\ t \ge 0)$$



globally correlated variables
finite q-variance;
q-Gaussian attractor

independent variables;
finite variance;
Gaussian attractor

independent variables;
divergent variance;
Levy attractor

M. Bologna, C. T. and P. Grigolini, Phys. Rev. E **62**, 2213 (2000)
C. T., Milan J. Math. **73**, 145 (2005)

q – PRODUCT:

L. Nivanen, A. Le Mehaute and Q.A. Wang, Rep. Math. Phys. **52**, 437 (2003)
E.P. Borges, Physica A **340**, 95 (2004)

The ***q*- product** is defined as follows:

$$x \otimes_q y \equiv \left[x^{1-q} + y^{1-q} - 1 \right]^{\frac{1}{1-q}}$$

$Properties:$

$i) \quad x \otimes_1 y = x\, y$

$ii) \quad \ln_q (x \otimes_q y) = \ln_q x + \ln_q y$ **(extensivity of Sq)**

$[whereas \;\; \ln_q (x\, y) = \ln_q x + \ln_q y + (1-q)(\ln_q x)(\ln_q y)]$

(nonadditivity of Sq)

q- GENERALIZED CENTRAL LIMIT THEOREM:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76,** 307 (2008)

q-Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x)\, dx = \int_{-\infty}^{\infty} e_q^{ix\xi[f(x)]^{q-1}} f(x)\, dx$$

$$(q \geq 1)$$

(nonlinear!)

$$q-FourierTransform\left[\frac{\sqrt{\beta}}{C_q}e_q^{-\beta\ t^2}\right]=e_{q_1}^{-\beta_1\ \omega^2}$$

where $$q_1=\frac{1+q}{3-q}$$

invertible

and $$\beta_1=\frac{3-q}{8\beta^{2-q}C_q^{2(1-q)}}\quad\Leftrightarrow\quad(\beta_1)^{\frac{1}{\sqrt{2-q}}}\ \beta^{\sqrt{2-q}}=\left[\frac{3-q}{8C_q^{2(1-q)}}\right]^{\frac{1}{\sqrt{2-q}}}\equiv K(q)$$

with $$C_q=\begin{cases}\dfrac{2\sqrt{\pi}\,\Gamma\left(\dfrac{1}{q-1}\right)}{(3-q)\sqrt{(1-q)}\,\Gamma\left(\dfrac{3-q}{2(1-q)}\right)} & if\quad q<1\\[2ex] \sqrt{\pi} & if\quad q=1\\[2ex] \dfrac{\sqrt{\pi}\,\Gamma\left(\dfrac{3-q}{2(q-1)}\right)}{\sqrt{q-1}\,\Gamma\left(\dfrac{1}{q-1}\right)} & if\quad 1<q<3\end{cases}$$

q- GENERALIZED CENTRAL LIMIT THEOREM:

q-independence:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76,** 307 (2008)

Two random variables X [*with density* $f_X(x)$] *and* Y [*with density* $f_Y(y)$] *having zero* $q-$*mean values are said* q-*independent if*

$$F_q[\mathrm{X+Y}](\xi) = F_q[\mathrm{X}](\xi) \otimes_{\frac{1+q}{3-q}} F_q[\mathrm{Y}](\xi) \quad ,$$

i.e., if

$$\int_{-\infty}^{\infty} dz\; e_q^{iz\xi} \otimes_q f_{X+Y}(z) = \left[\int_{-\infty}^{\infty} dx\; e_q^{ix\xi} \otimes_q f_X(x)\right] \otimes_{(1+q)/(3-q)} \left[\int_{-\infty}^{\infty} dy\; e_q^{iy\xi} \otimes_q f_Y(y)\right],$$

with

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\; h(x,y)\; \delta(x+y-z) = \int_{-\infty}^{\infty} dx\; h(x, z-x) = \int_{-\infty}^{\infty} dy\; h(z-y, y)$$

where $h(x,y)$ *is the joint density.*

$$q\text{-}\mathit{independence\ means} \begin{cases} \mathit{independence} & \mathit{if}\;\; q=1\,,\; \mathit{i.e.,}\;\; h(x,y) = f_X(x)\, f_Y(y) \\ \mathit{global\ correlation} & \mathit{if}\;\; q \neq 1\,,\; \mathit{i.e.,}\;\; h(x,y) \neq f_X(x)\, f_Y(y) \end{cases}$$

A random variable X is said to have a (q,α)*-stable distribution* $L_{q,\alpha}(x)$

if its q-Fourier transform has the form $a\, e_{q_1}^{-b\,|\xi|^{\alpha}}$

$[a>0,\ b>0,\ 0<\alpha\leq 2,\ q_1 \equiv (q+1)/(3-q)]$

i.e., if

$$F_q[L_{q,\alpha}](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q L_{q,\alpha}(x)\, dx = \int_{-\infty}^{\infty} e_q^{\frac{ix\xi}{[L_{q,\alpha}(x)]^{1-q}}} L_{q,\alpha}(x)\, dx = a\, e_{q_1}^{-b\,|\xi|^{\alpha}}$$

$L_{1,2}(x) \equiv G(x)$ [*Gaussian*]

$L_{1,\alpha}(x) \equiv L_{\alpha}(x)$ [α-*stable Levy distribution*]

$L_{q,2}(x) \equiv G_q(x)$ [*q-Gaussian*]

$L_{q,\alpha}(x)$ [(q,α)-*stable distribution*]

S. Umarov, C. T., M. Gell-Mann and S. Steinberg

cond-mat/0606038v2 and cond-mat/0606040v2 (2008)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ - *scaled attractor* $\mathbb{F}(x)$ *when summing* $N \to \infty$ q - *independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \dfrac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x\rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x\rvert^{2/(q-1)} & \text{if } \lvert x\rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x\rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x\rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x\rvert^{1+\alpha} \\ \quad \text{if } \lvert x\rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, *with same* $\lvert x\rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\,\alpha}(x) \sim C^*_{q,\alpha} / \lvert x\rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (\textit{intermediate regime}) \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\,2}(x) \sim C^L_{q,\alpha} / \lvert x\rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \quad (\textit{distant regime}) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg cond-mat/0606038v2 and cond-mat/0606040v2 (2008)

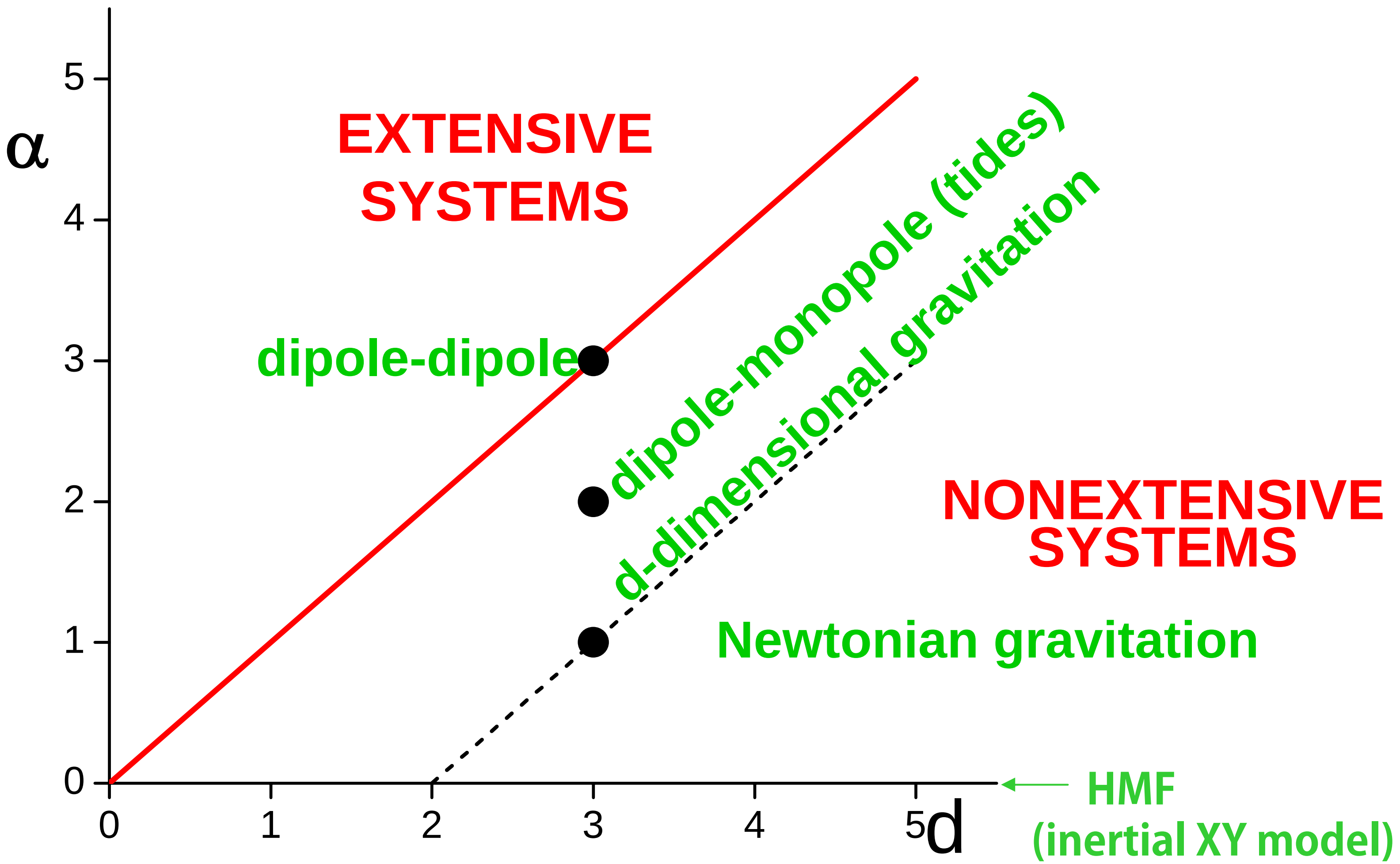
SOME EXPERIMENTAL, OBSERVATIONAL AND COMPUTATIONAL VERIFICATIONS AND APPLICATIONS

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(\vec{r}) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0, \quad \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*



d-DIMENSIONAL CLASSICAL INERTIAL *XY* FERROMAGNET:

(We illustrate with the *XY* (i.e., *n=2*) model; the argument holds however true for any *n>1* and any *d*-dimensional Bravais lattice)

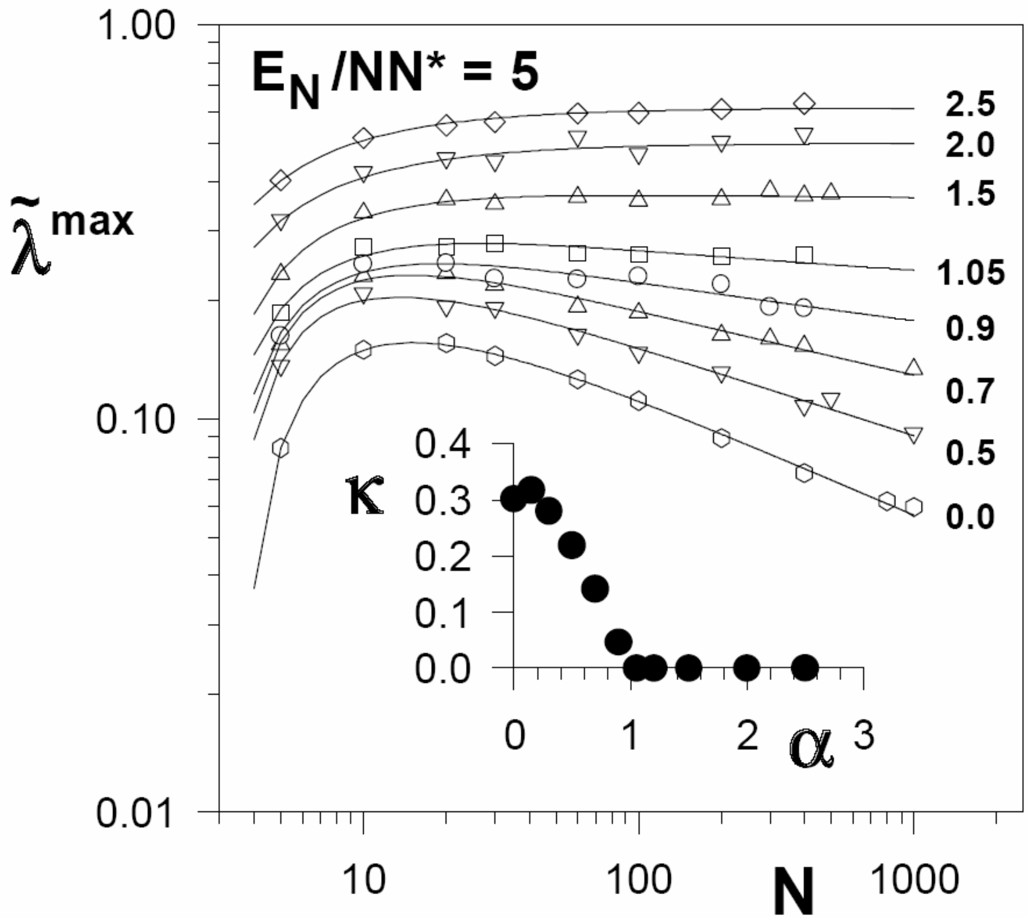
$$H = K + V = \frac{1}{2I}\sum_{i=1}^{N} L_i^2 + \frac{J}{\mathfrak{A}}\sum_{i,j}\frac{1-\cos(\vartheta_i - \vartheta_j)}{r_{ij}^{\alpha}} \qquad (I > 0,\ J > 0)$$

$$with\ \ \mathfrak{A} \equiv \sum_{j=1}^{N} r_{ij}^{-\alpha} \propto \begin{cases} N^{1-\alpha/d} & if \quad 0 \le \alpha/d < 1 \\ \ln N & if \quad \alpha/d = 1 \\ constant & if \quad \alpha/d > 1 \end{cases}$$

and periodic boundary conditions.

$[The\ HMF\ \ model\ corresponds\ to\ \alpha/d = 0]$

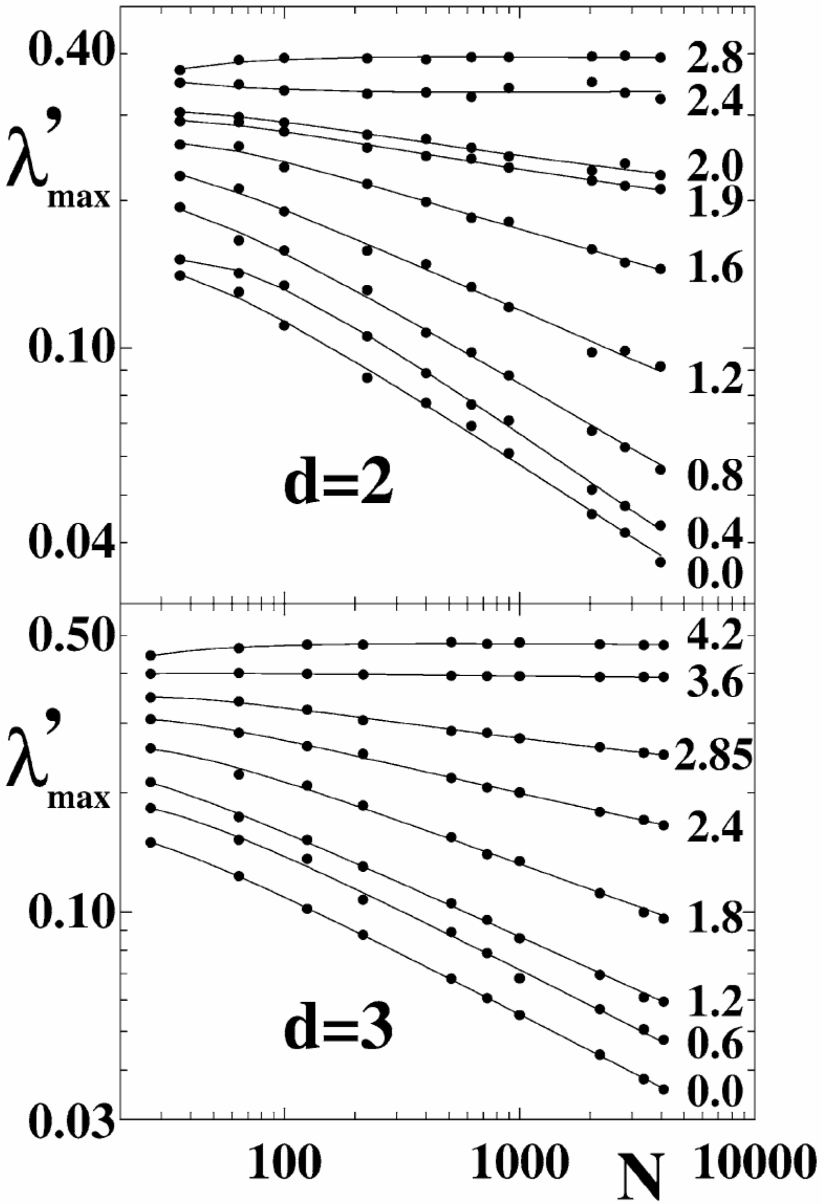
C. Anteneodo and C. T., Phys Rev Lett **80**, 5313 (1998)

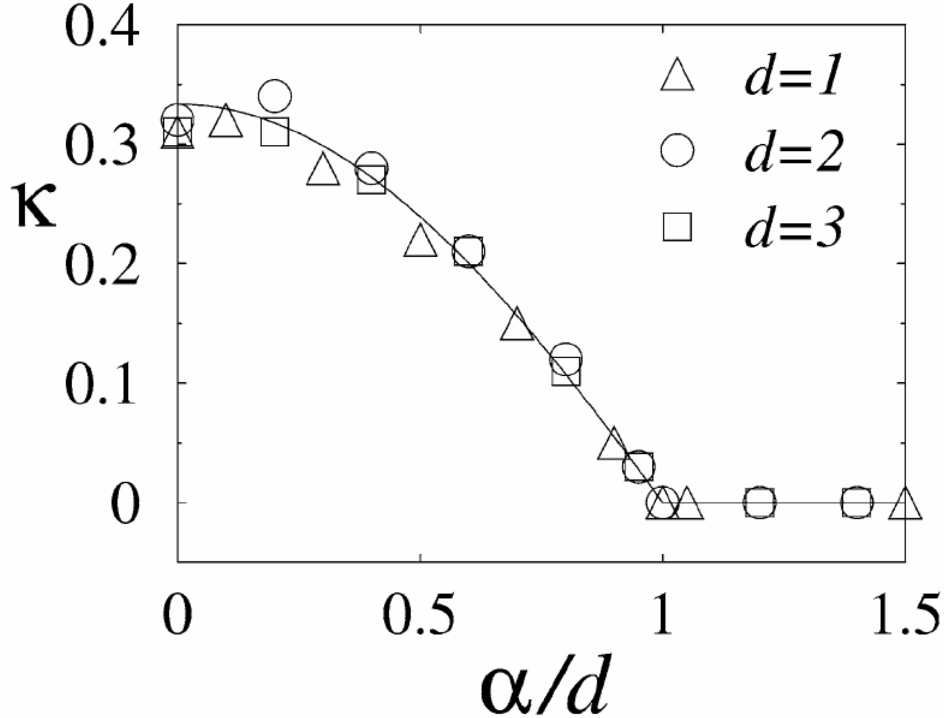


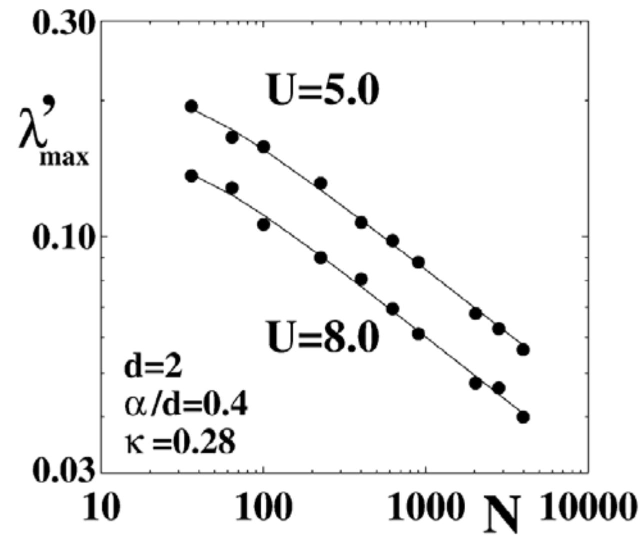
$$\lambda^{max} \sim \frac{1}{N^{\kappa(\alpha)}}$$

FIG. 3. $\tilde{\lambda}_N^{\max}$ versus N (log-log plot) for typical values of α and $\frac{E_N}{NN^*} = 5$. The full lines are the best fittings with the forms $(a - \frac{b}{N})/(N^*)^c$. Consequently, $\tilde{\lambda}_N^{\max} \propto N^{-\kappa(\alpha)}$ where $\kappa(\alpha) = (1-\alpha)c$ for $0 \leq \alpha < 1$ and $\kappa(\alpha) = 0$ for $\alpha > 1$; for $\alpha = 1$, $\tilde{\lambda}_N^{\max}$ is expected to vanish as a power of $1/\ln N$. Inset: κ versus α (related random matrices arguments will be detailed elsewhere).

C. Anteneodo and C. T., Phys Rev Lett **80**, 5313 (1998)

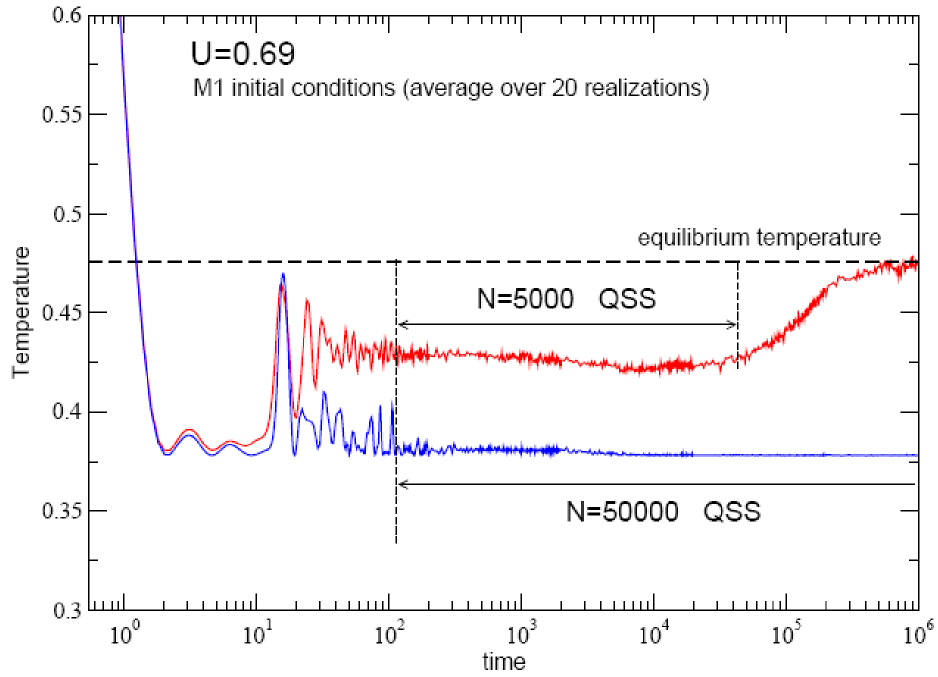


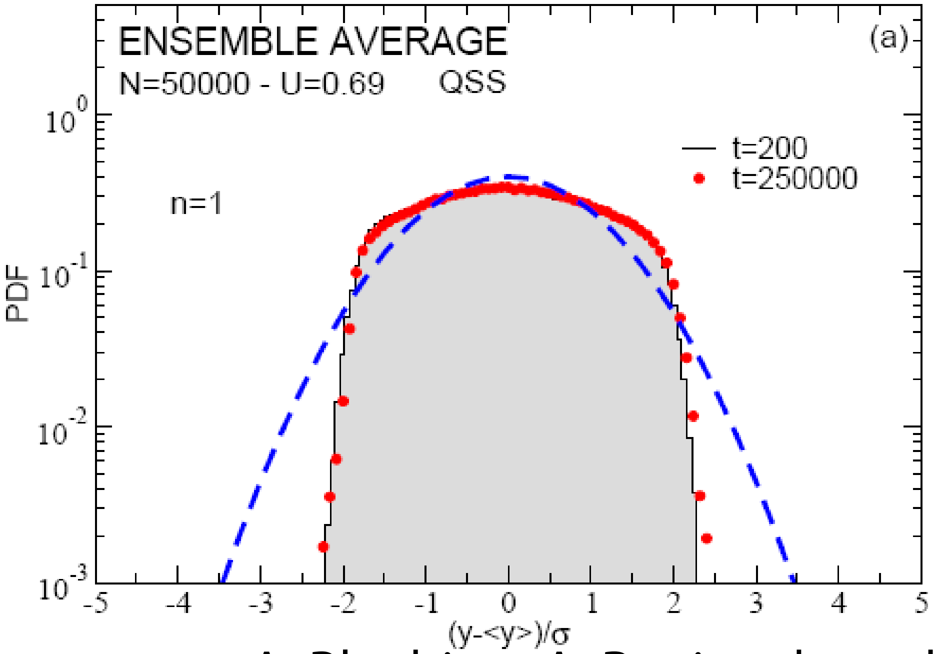


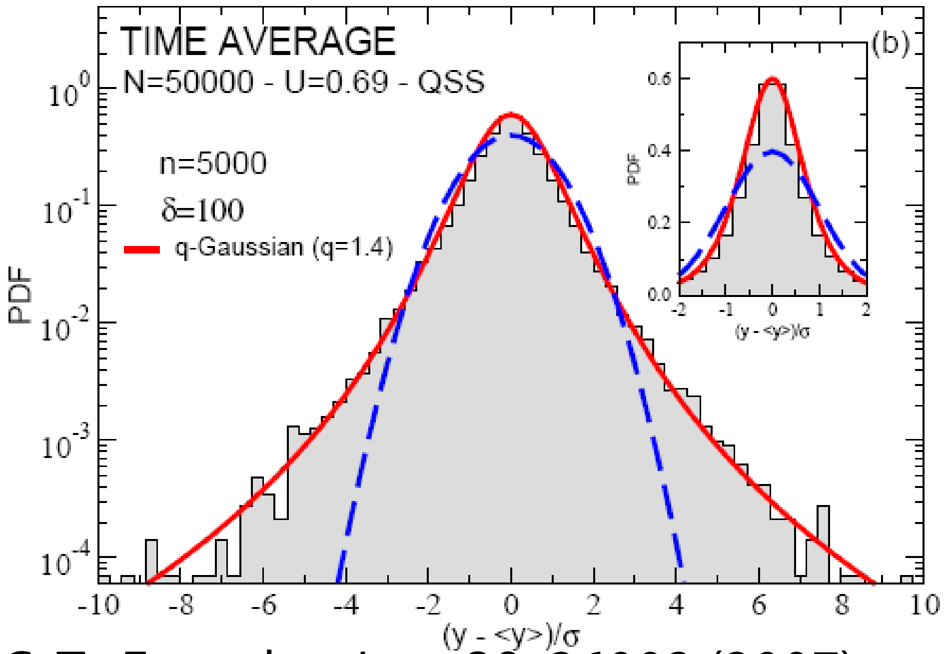


A. Campa, A. Giansanti, D. Moroni and C. T., Phys Lett A **286**, 251 (2001)

HMF MODEL

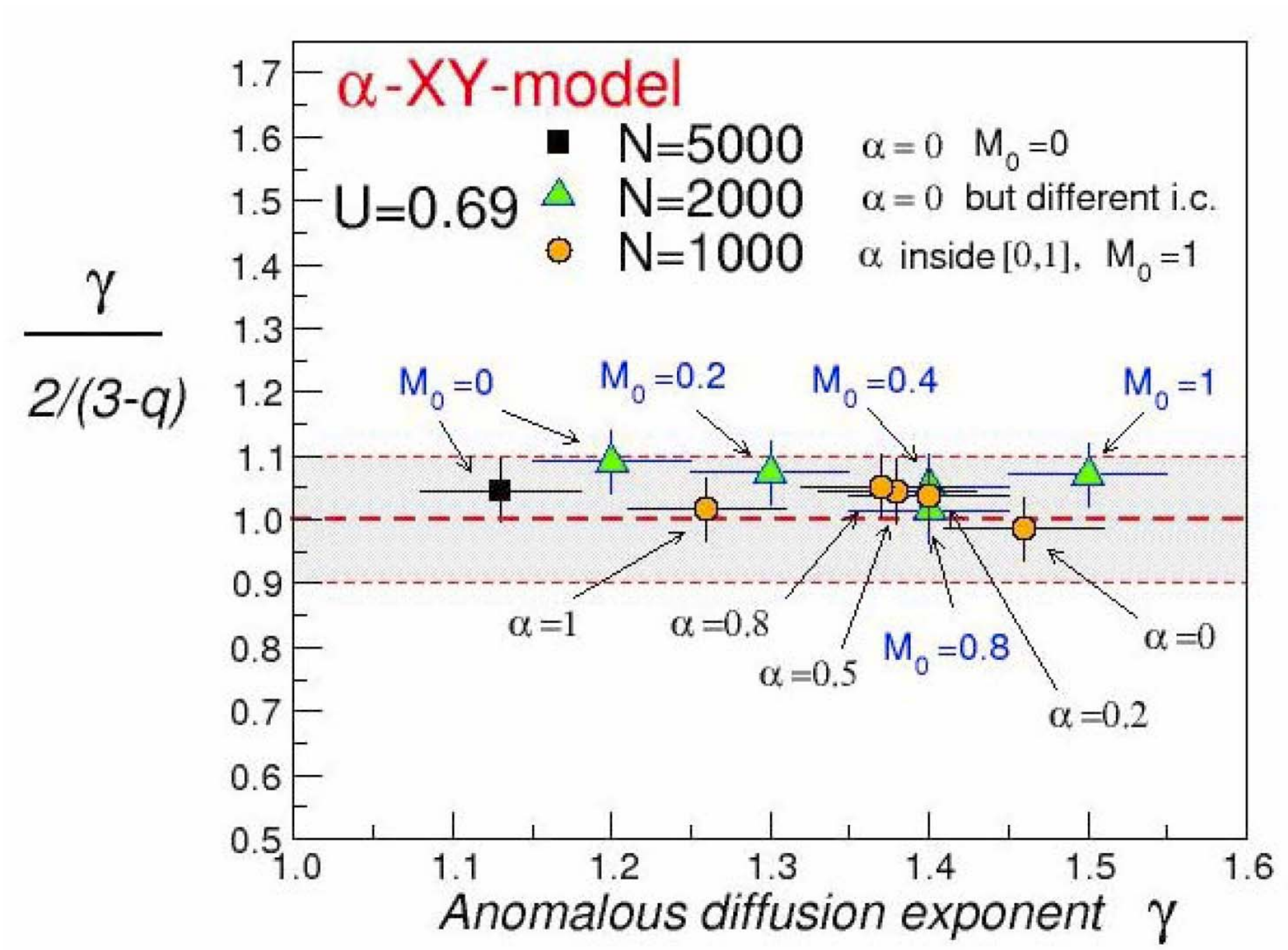






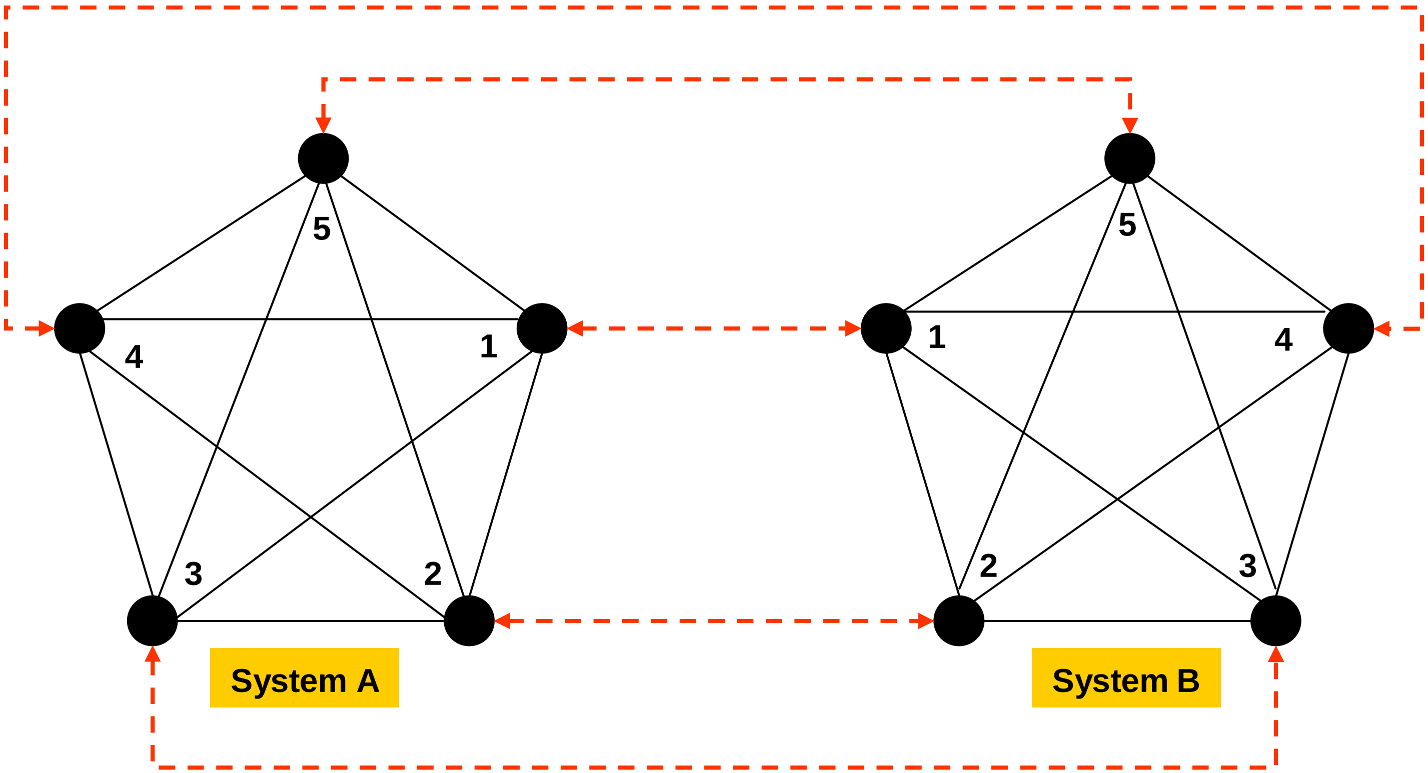
A. Pluchino, A. Rapisarda and C. T., Europhys Lett **80**, 26002 (2007)

XY FERROMAGNET WITH LONG-RANGE INTERACTIONS:



A Rapisarda and A Pluchino, Europhys News **36**, 202 (2005) [European Physical Society]

THERMAL CONTACT BETWEEN SYSTEMS A AND B:

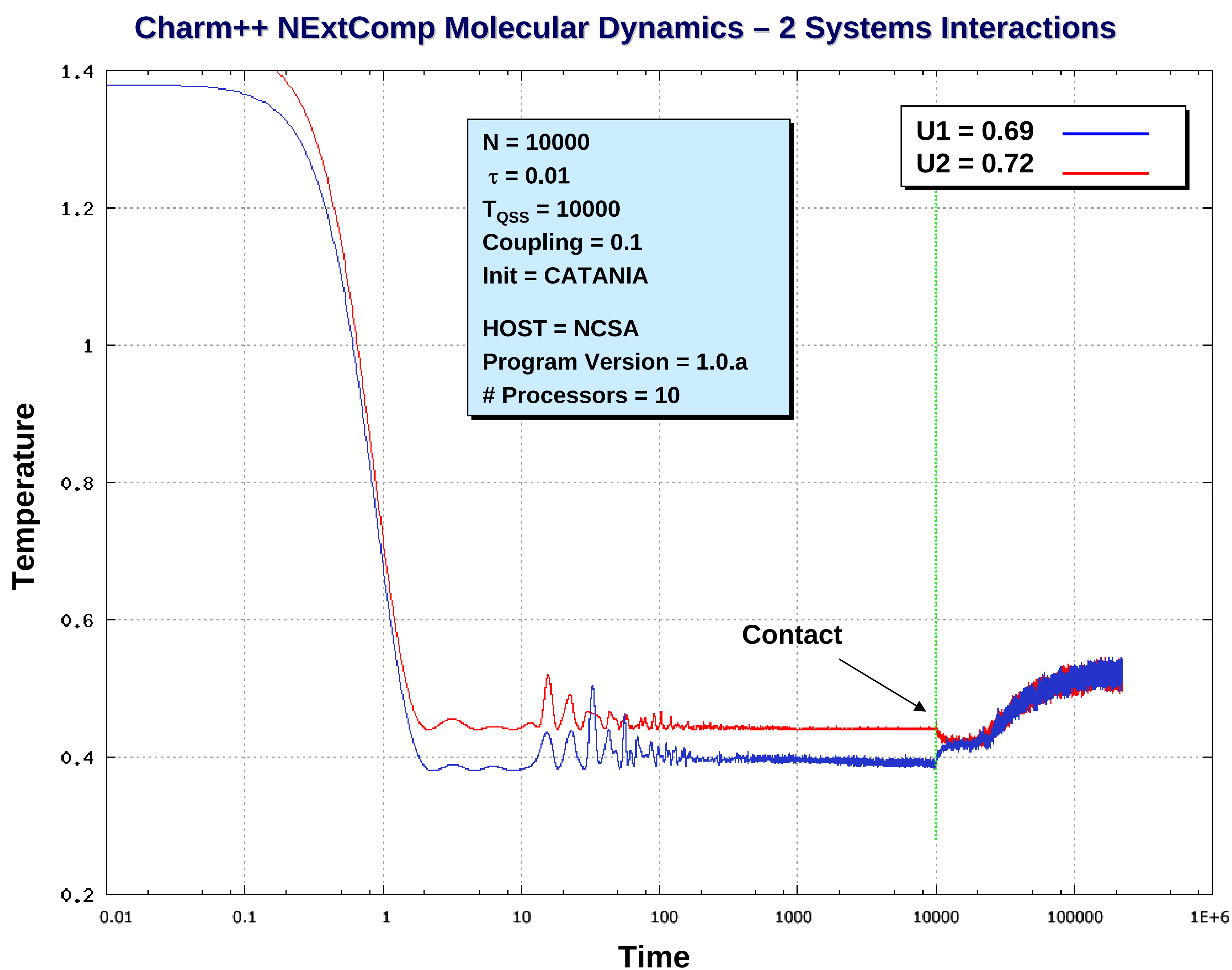


=

$$H = \sum_{i=1}^{N} \frac{(L_i^A)^2}{2} + \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} [1 - \cos(\theta_i^A - \theta_j^A)]$$

$$+ \sum_{i=1}^{N} \frac{(L_i^B)^2}{2} + \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} [1 - \cos(\theta_i^B - \theta_j^B)]$$

$$+ \; l \sum_{k=1}^{N} [1 - \cos(\theta_k^A - \theta_k^B)]$$


Charm++ NExtComp Molecular Dynamics – 2 Systems Interactions
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T_QSS = 10000
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Init = CATANIA
HOST = NCSA
Program Version = 1.0.a
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U2 = 0.72
Contact
Temperature
Time
1.4
1.2
1
0.8
0.6
0.4
0.2
0.01
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1000
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100000
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PHYSICAL REVIEW E **79**, 040103(R) (2009)

Analysis of self-organized criticality in Ehrenfest's dog-flea model

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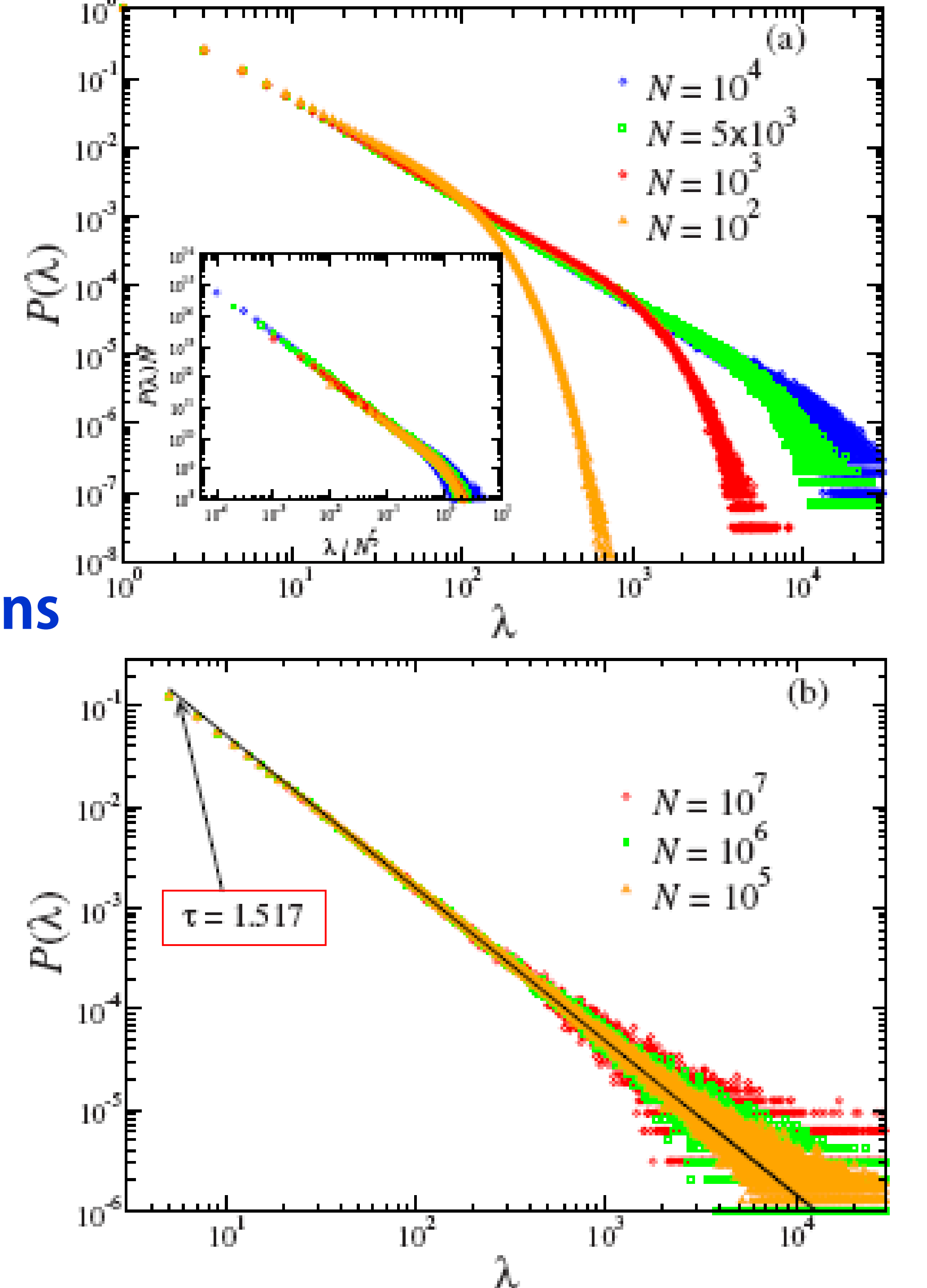
[2]*Division of Statistical Mechanics and Complexity, Institute of Theoretical and Applied Physics (ITAP) Kaygiseki Mevkii, 48740 Turunc, Mugla, Turkey*

(Received 8 January 2009; published 23 April 2009)

The self-organized criticality in Ehrenfest's historical dog-flea model is analyzed by simulating the underlying stochastic process. The fluctuations around the thermal equilibrium in the model are treated as avalanches. We show that the distributions for the fluctuation length differences at subsequent time steps are in the shape of a q-Gaussian (the distribution which is obtained naturally in the context of nonextensive statistical mechanics) if one avoids the finite-size effects by increasing the system size. We provide clear numerical evidence that the relation between the exponent τ of avalanche size distribution obtained by maximum-likelihood estimation and the q value of appropriate q-Gaussian obeys the analytical result recently introduced by Caruso *et al.* [Phys. Rev. E **75**, 055101(R) (2007)]. This allows us to determine the value of q-parameter *a priori* from one of the well-known exponents of such dynamical systems.

Fluctuation length distributions



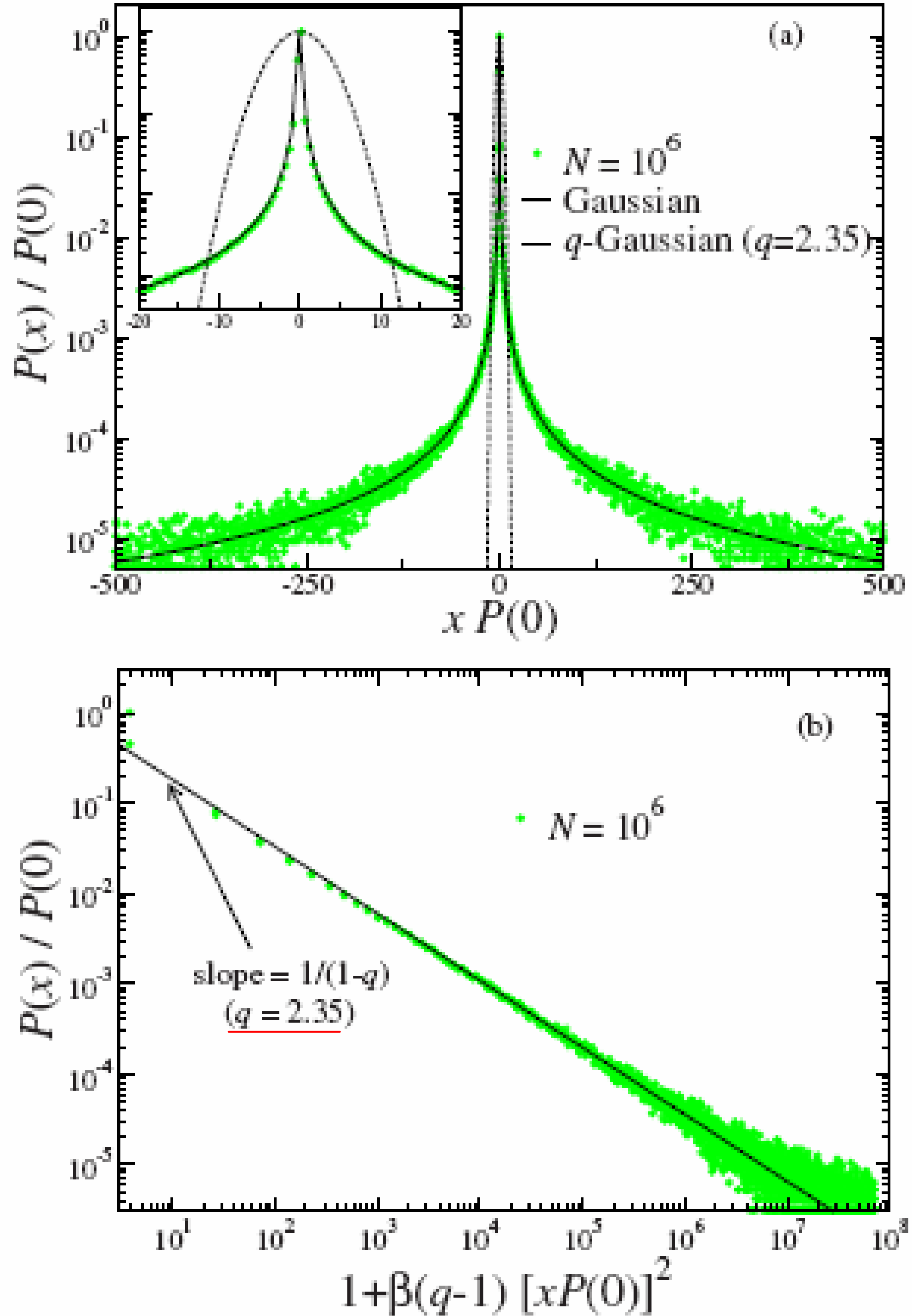
B. Bakar and U. Tirnakli Phys Rev E **79**, 040103(R) (2009)

Distribution of returns

$$q = e^{1.19\tau^{-0.795}}$$

2.35

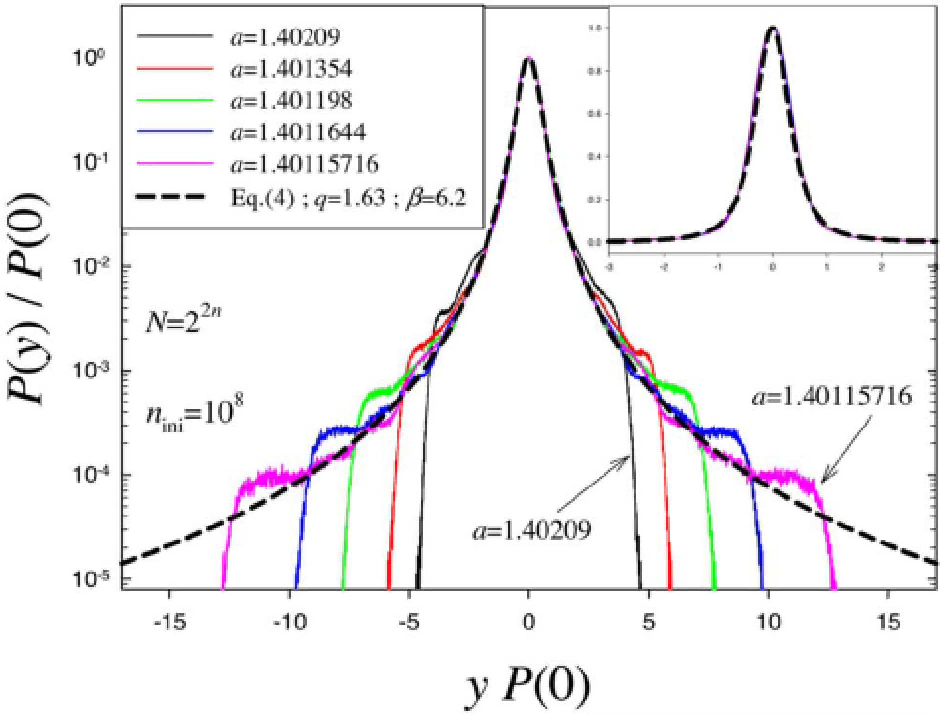
1.517



B. Bakar and U. Tirnakli

Phys Rev E **79**, 040103(R) (2009)

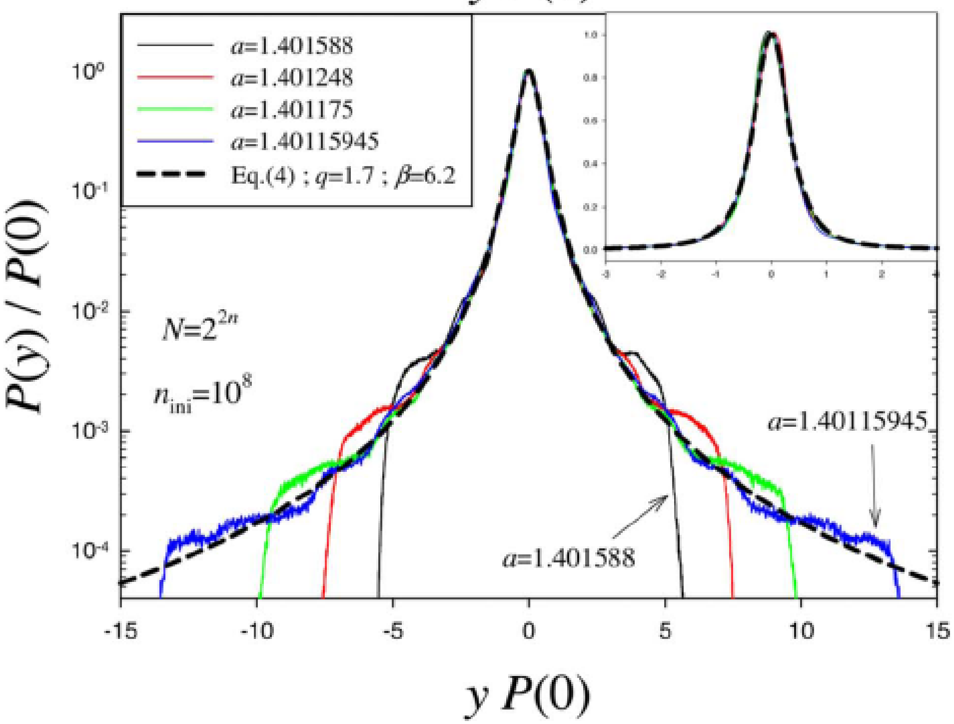
LOGISTIC MAP: EDGE OF CHAOS



odd 2n

q=1.63

beta=6.2



even 2n

q=1.70

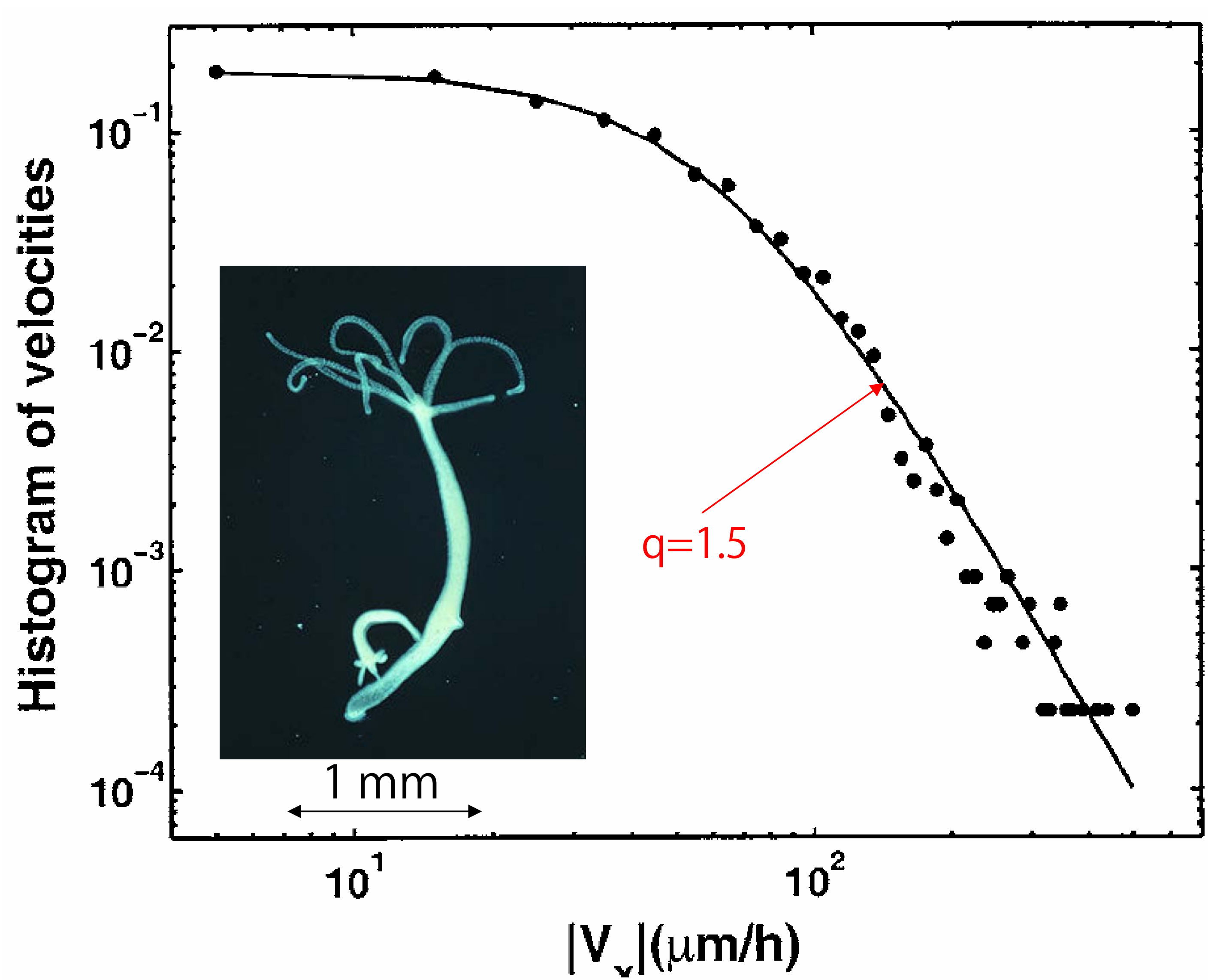
beta=6.2

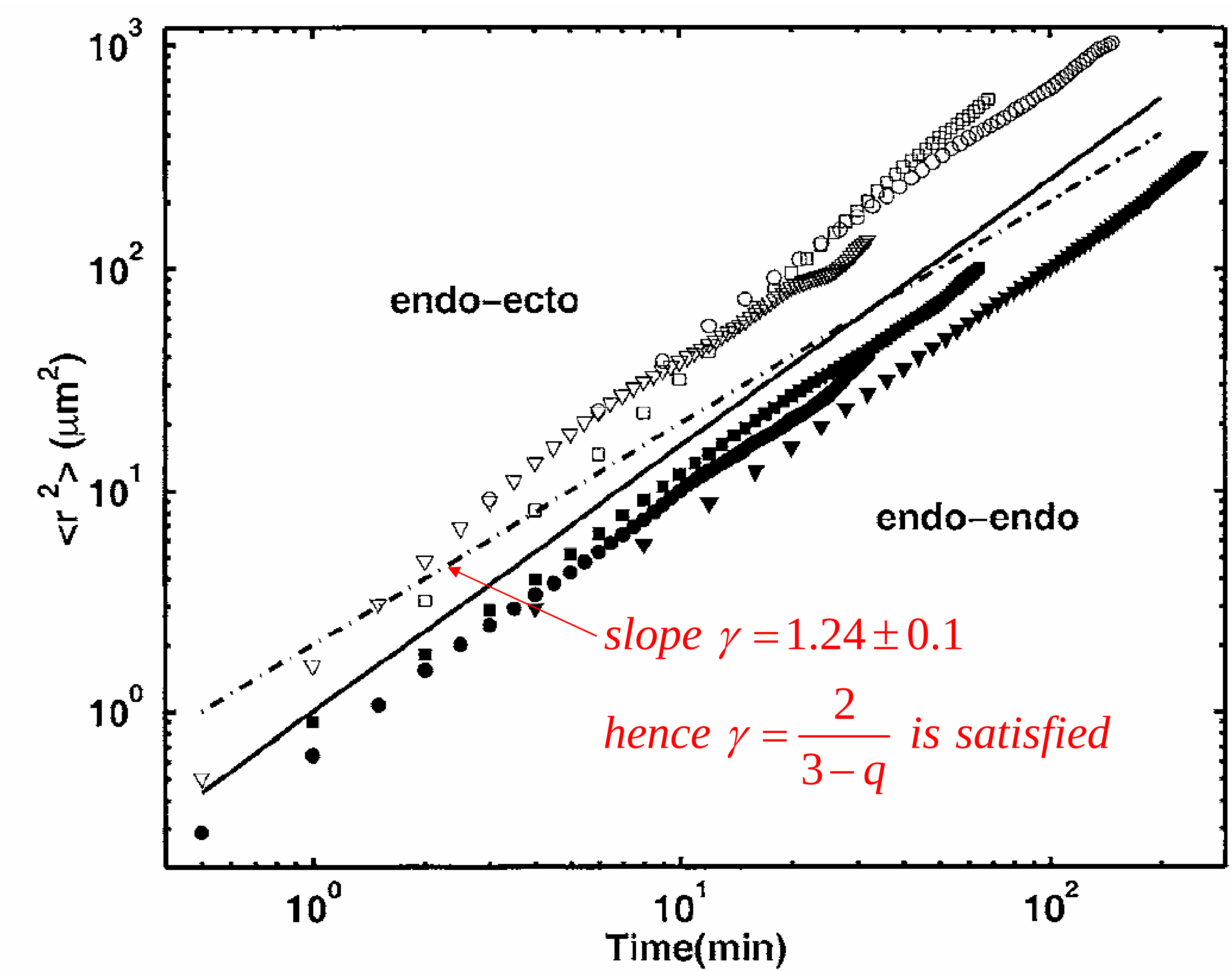
U. Tirnakli, C. Beck and C. T.
Phys Rev E **75**, 040106(R) (2007)

U. Tirnakli, C. T. and C. Beck
Phys Rev E **79**, 056209 (2009)

Hydra viridissima:

A Upadhyaya, J-P Rieu, JA Glazier and Y Sawada, Physica A **293**, 549 (2001)





A Upadhyaya, J-P Rieu, JA Glazier and Y Sawada, Physica A **293**, 549 (2001)

COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

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PHYSICAL REVIEW A **67**, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

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(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A **245**, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a q-Gaussian;

(ii) $$q = 1 + \frac{44 E_R}{U_0}$$ where $E_R \equiv$ recoil energy

$U_0 \equiv$ potential depth

Experimental and computational verifications in optical lattices:

PRL **96,** 110601 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Tunable Tsallis Distributions in Dissipative Optical Lattices

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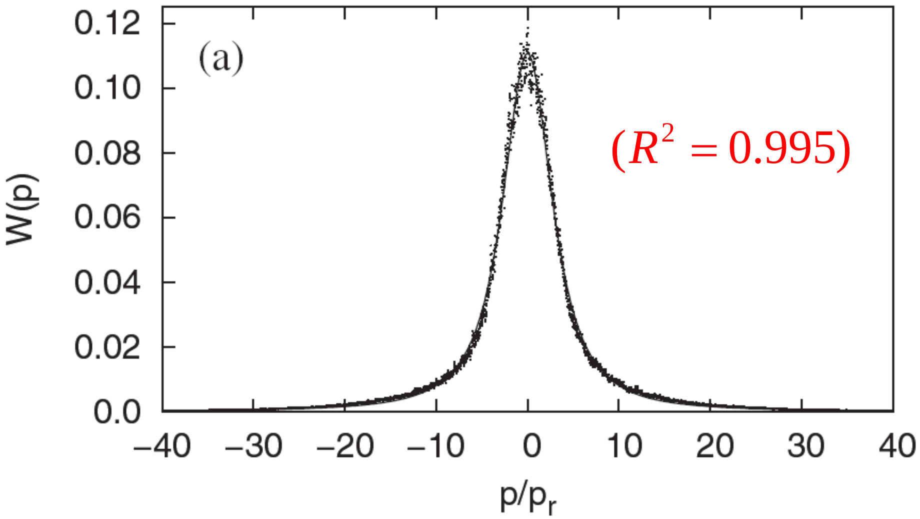
Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

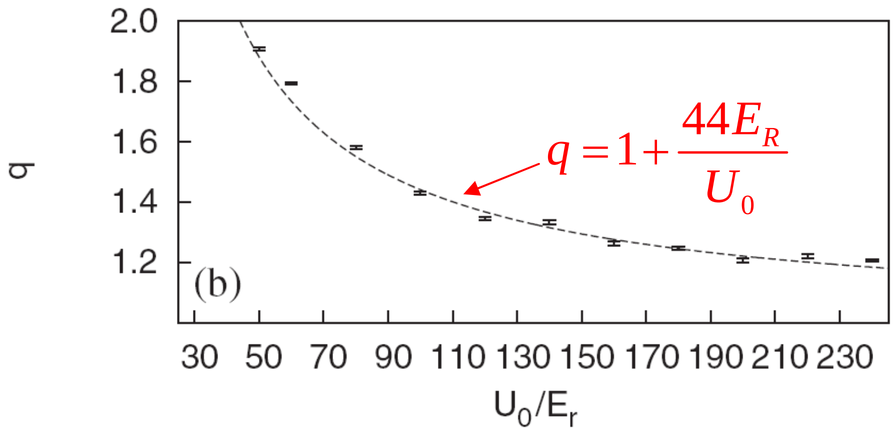
(Received 10 January 2006; published 24 March 2006)

We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

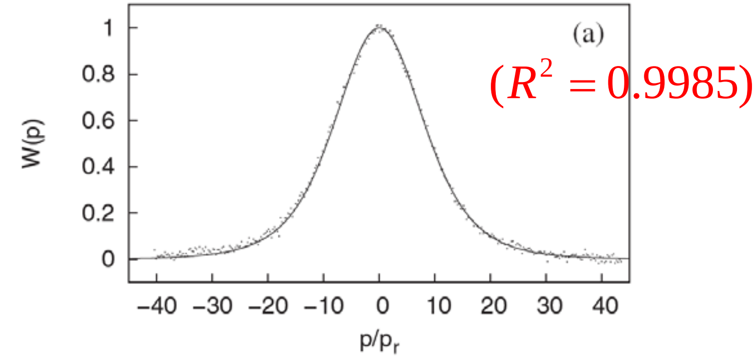
Experimental and computational verifications

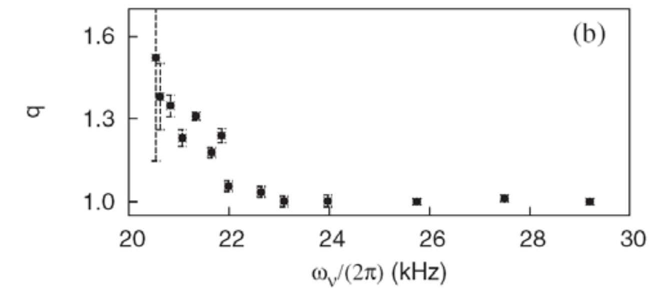
by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)

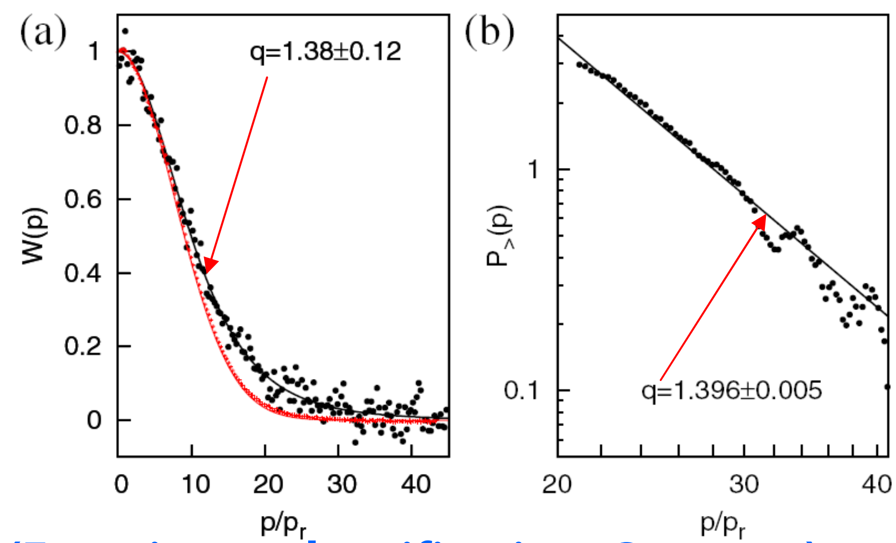




(Computational verification: quantum Monte Carlo simulations)







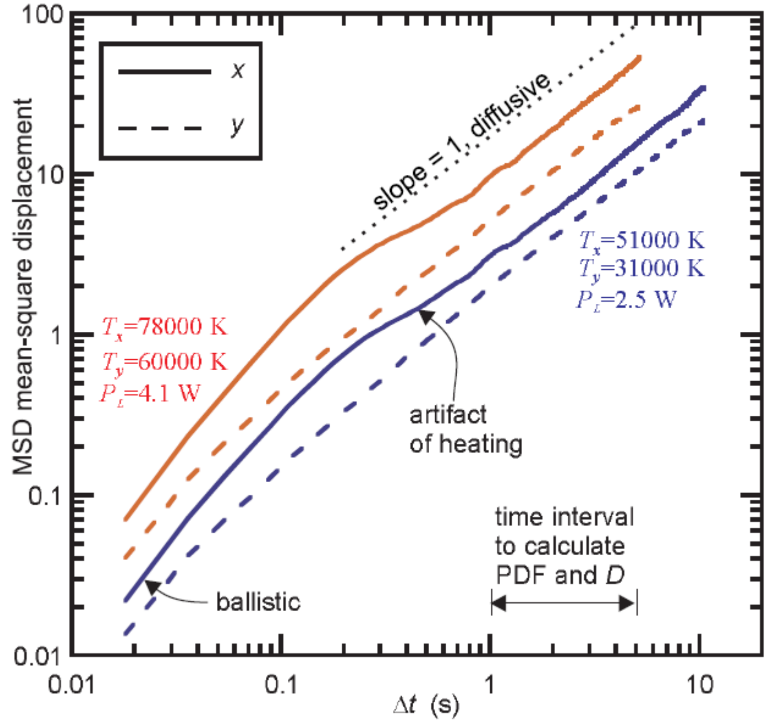
(Experimental verification: Cs atoms)

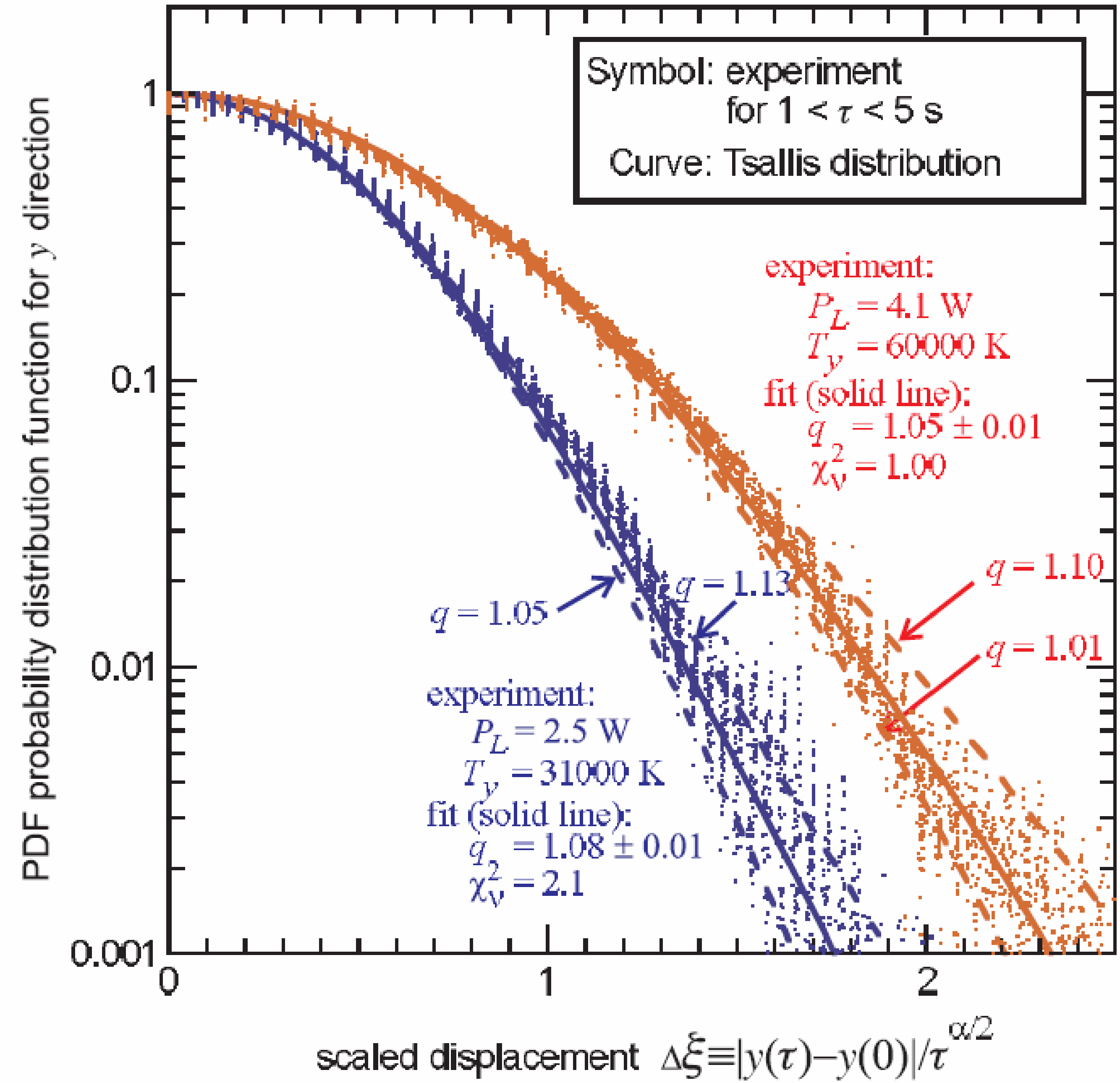
Superdiffusion and Non-Gaussian Statistics in a Driven-Dissipative 2D Dusty Plasma

Bin Liu and J. Goree
Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242, USA
(Received 1 June 2007; published 6 February 2008)

Anomalous diffusion and non-Gaussian statistics are detected experimentally in a two-dimensional driven-dissipative system. A single-layer dusty plasma suspension with a Yukawa interaction and frictional dissipation is heated with laser radiation pressure to yield a structure with liquid ordering. Analyzing the time series for mean-square displacement, superdiffusion is detected at a low but statistically significant level over a wide range of temperatures. The probability distribution function fits a Tsallis distribution, yielding q, a measure of nonextensivity for non-Gaussian statistics.

$$\langle r^2 \rangle \propto t^{\alpha}$$





B. Liu and J. Goree, Phys Rev Lett **100**, 055003 (2008)

LASER COOLING:

PRL **102,** 063001 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

Power-Law Distributions for a Trapped Ion Interacting with a Classical Buffer Gas

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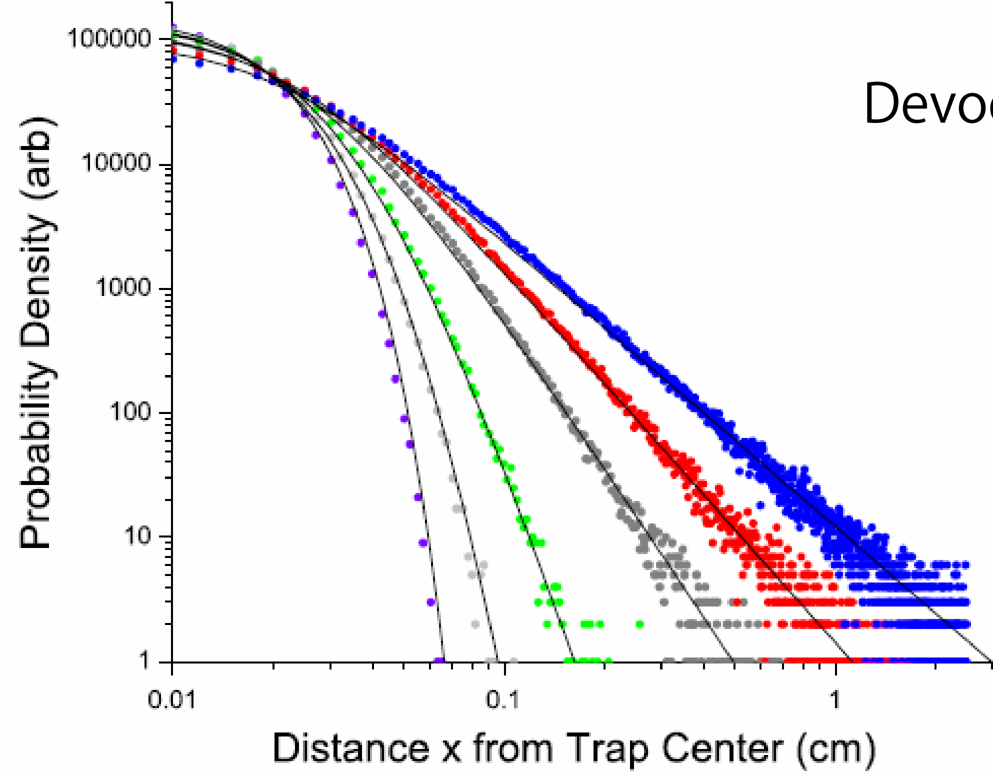
Physics Department, Stanford University, Stanford, California 94305, USA
(Received 3 November 2008; published 10 February 2009)

Classical collisions with an ideal gas generate non-Maxwellian distribution functions for a single ion in a radio frequency ion trap. The distributions have power-law tails whose exponent depends on the ratio of buffer gas to ion mass. This provides a statistical explanation for the previously observed transition from cooling to heating. Monte Carlo results approximate a Tsallis distribution over a wide range of parameters and have *ab initio* agreement with experiment.



Devoe, Phys Rev Lett **102** (2009) 063001

$$T(x) = \frac{T(0)}{\left[1 + (q-1)\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}}$$

FIG. 1 (color online). Monte Carlo distributions for a single $^{136}Ba^{+}$ ion cooled by six different buffer gases at 300 K ranging from $m_B = 4$ (left) to $m_B = 200$ (right). Note the evolution from Gaussian to power law (straight line) as the mass increases. The solid lines are Tsallis functions [Eq. (7)] with fixed $\sigma = 0.0185$ cm and the exponents of Table I.

TABLE I. Tsallis parameters n and q_T fit from Fig. 1.

Buffer gas	m_I/m_B	n	q_T
He	34.5	>60	1.03
Ar	3.40	8.2	1.12
Kr	1.70	3.8	1.26
Xe	1.0	1.98	1.51
170	0.80	1.50	1.80
200	0.68	1.15	1.87

SPIN RELAXATION IN SPIN GLASSES (NEUTRON SPIN ECHO):

PRL **102,** 097202 (2009) PHYSICAL REVIEW LETTERS week ending 6 MARCH 2009

Generalized Spin-Glass Relaxation

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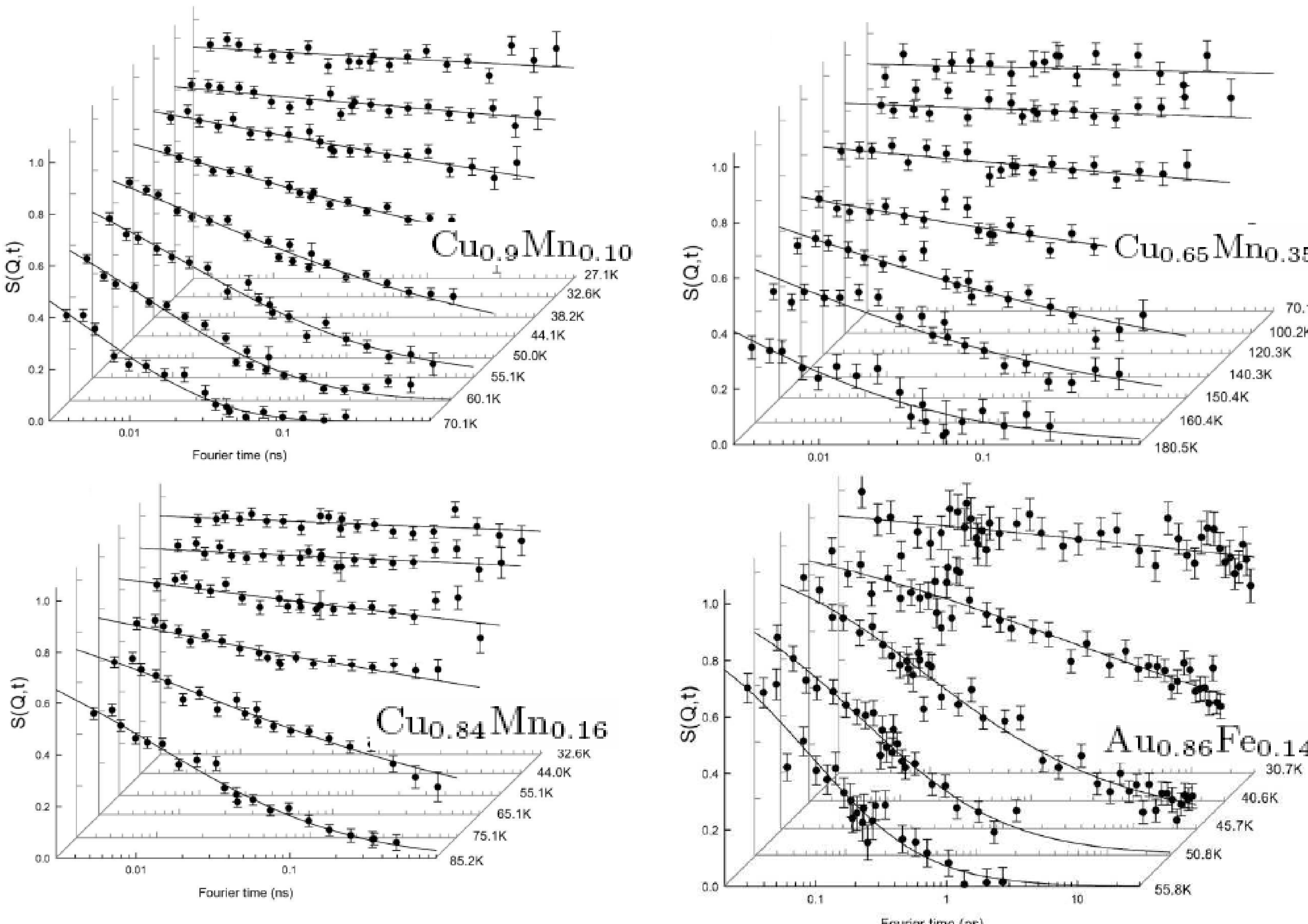
[1]*School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[2]*School of Applied Sciences, University of Huddersfield, Huddersfield HD1 3DH, United Kingdom*
[3]*Helmholtz Center Berlin for Materials and Energy, Glienickerstrasse 100, 14109, Berlin, Germany*
[4]*Institut Laue Langevin, 6 rue Jules Horowitz, 38000 Grenoble, France*

(Received 18 July 2008; published 4 March 2009)

Spin relaxation close to the glass temperature of CuMn and AuFe spin glasses is shown, by neutron spin echo, to follow a generalized exponential function which explicitly introduces hierarchically constrained dynamics and macroscopic interactions. The interaction parameter is directly related to the normalized Tsallis nonextensive entropy parameter q and exhibits universal scaling with reduced temperature. At the glass temperature $q = 5/3$ corresponding, within Tsallis' q statistics, to a mathematically defined critical value for the onset of strong disorder and nonlinear dynamics.



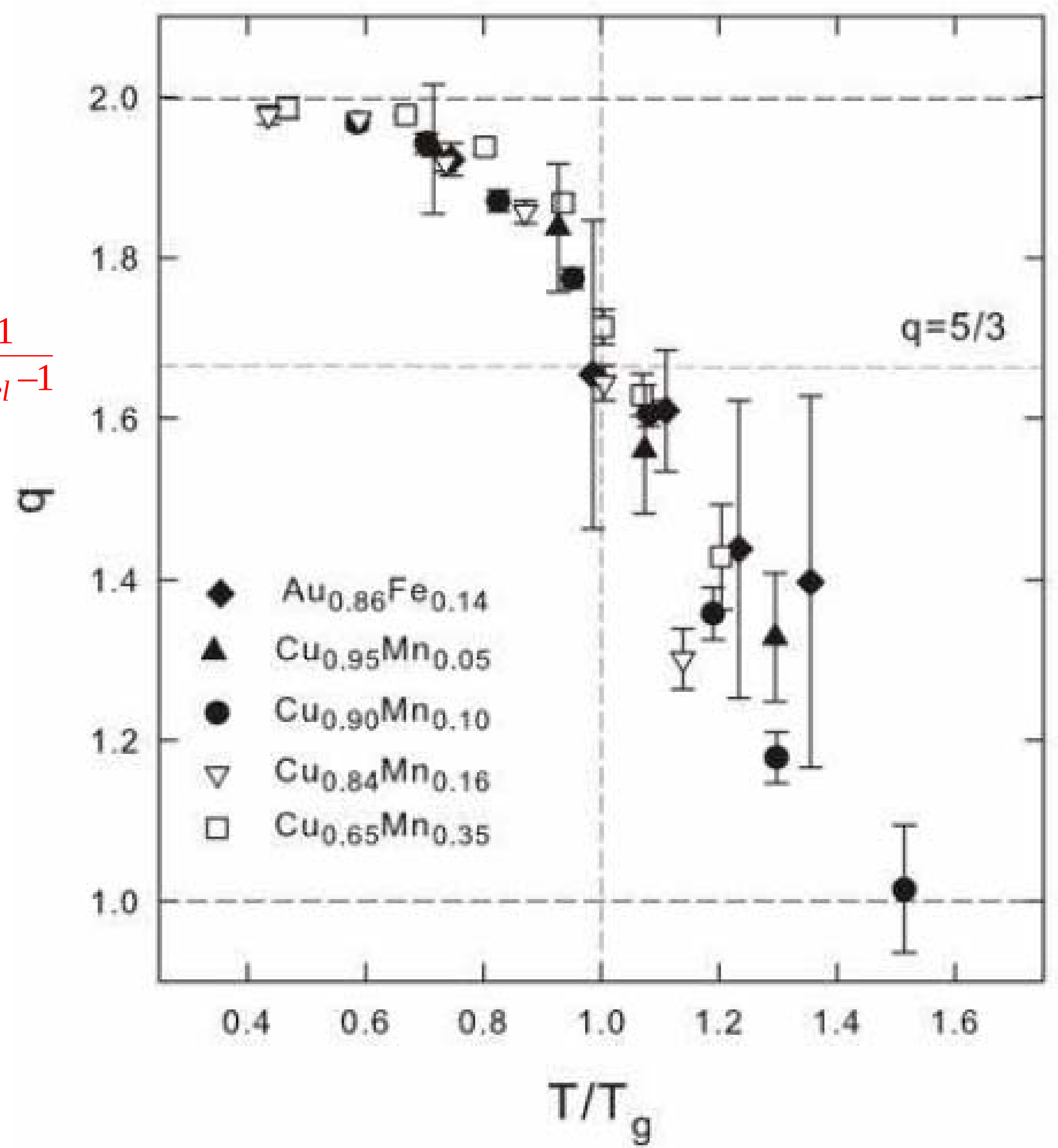
Pickup, Cywinski, Pappas, Farago, Fouquet, Phys Rev Lett **102** (2009) 097202

SPIN RELAXATION IN SPIN GLASSES (NEUTRON SPIN ECHO):

$$\phi(t) = \left[1 + \frac{q-1}{2-q} \left(\frac{t}{\tau} \right)^{\beta} \right]^{-\frac{2-q}{q-1}}$$

$$\equiv \left[1 + (q_{rel} - 1) \left(\frac{t}{\tau} \right)^{\beta} \right]^{-\frac{1}{q_{rel} - 1}}$$

$$q_{rel} \equiv \frac{1}{2-q}$$

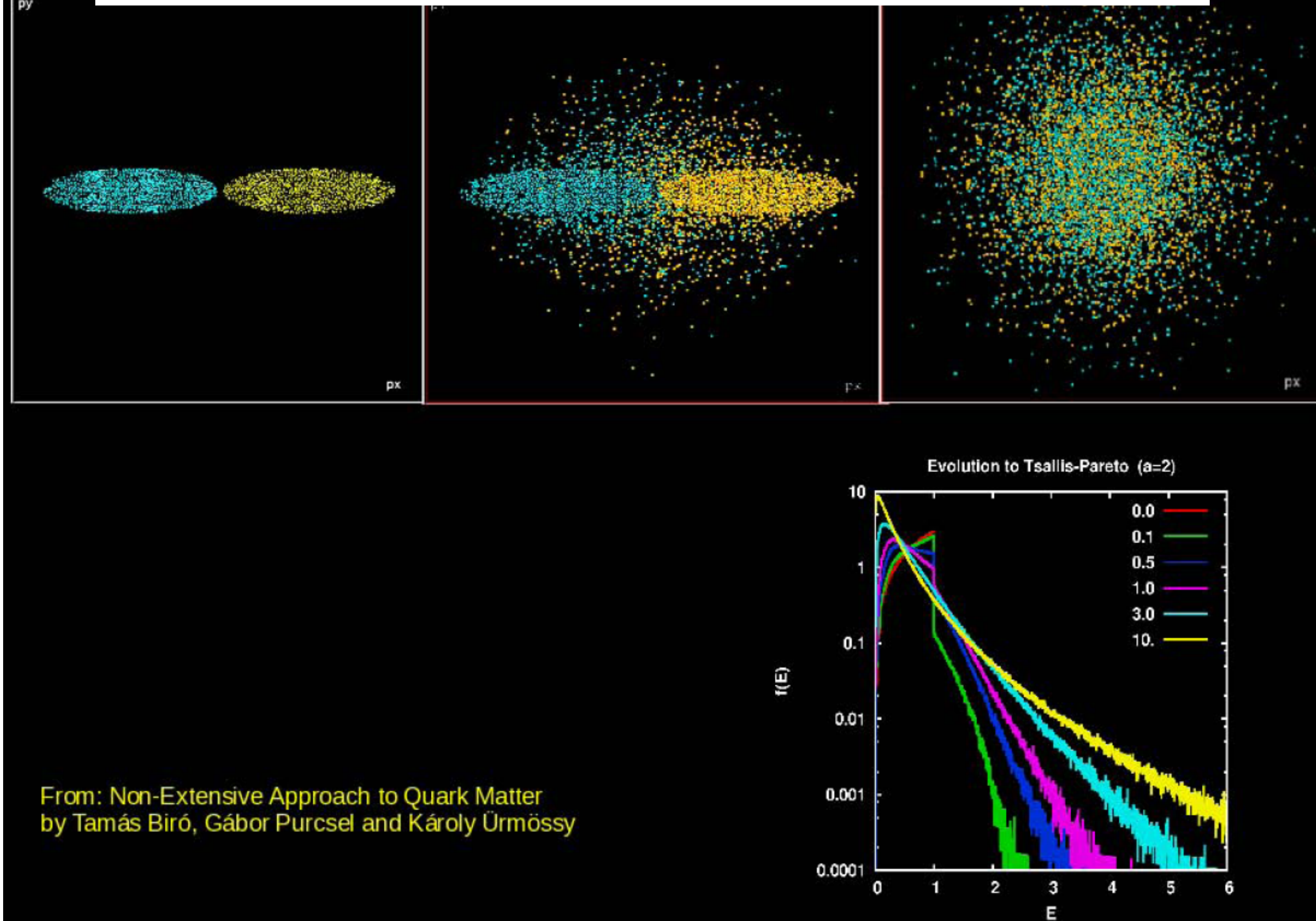


Pickup, Cywinski, Pappas, Farago, Fouquet, Phys Rev Lett **102** (2009) 097202

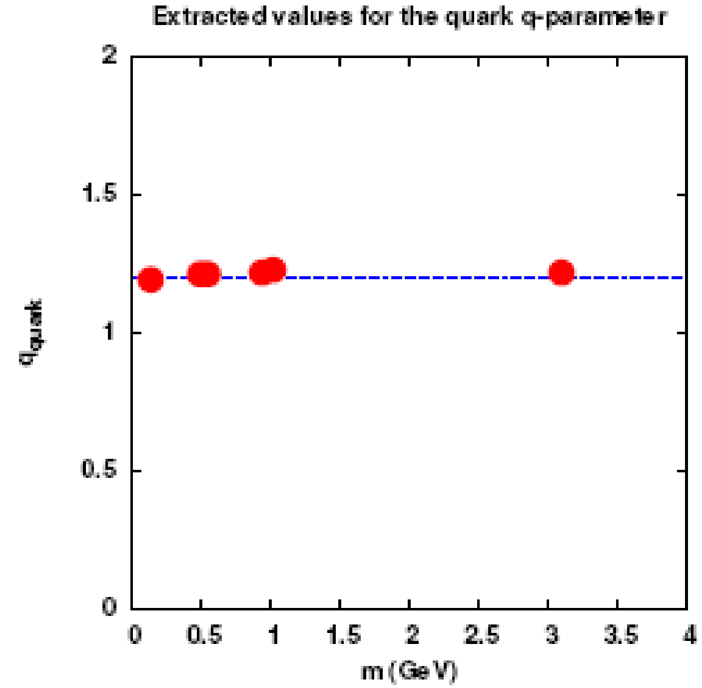
Non-Extensive Approach to Quark Matter

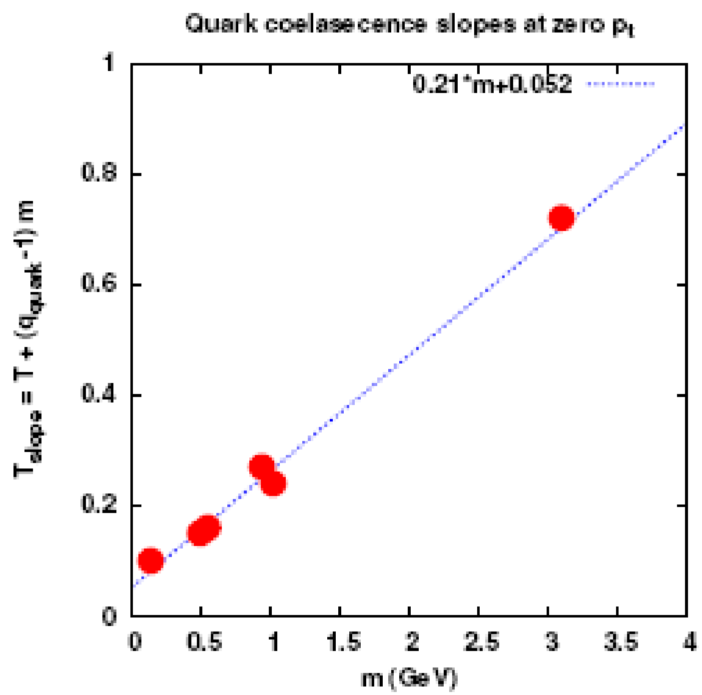
Tamás S. Biró[1], Gábor Purcsel[1] and Károly Ürmössy[1]

KFKI Research Institute for Particle and Nuclear Physics



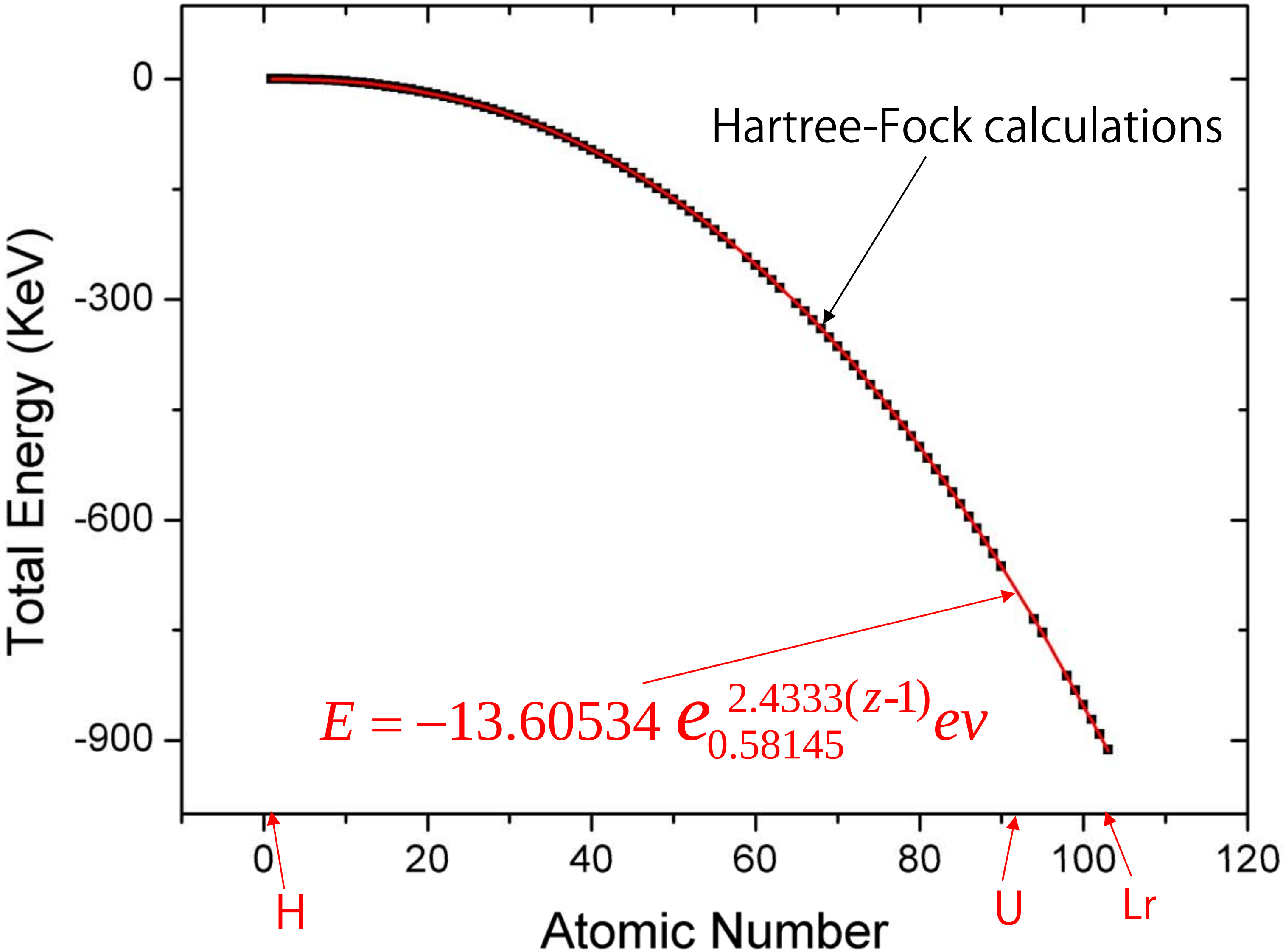
MODEL OF QUARK COALESCENCE AT A SUDDEN HADRON FORMATION





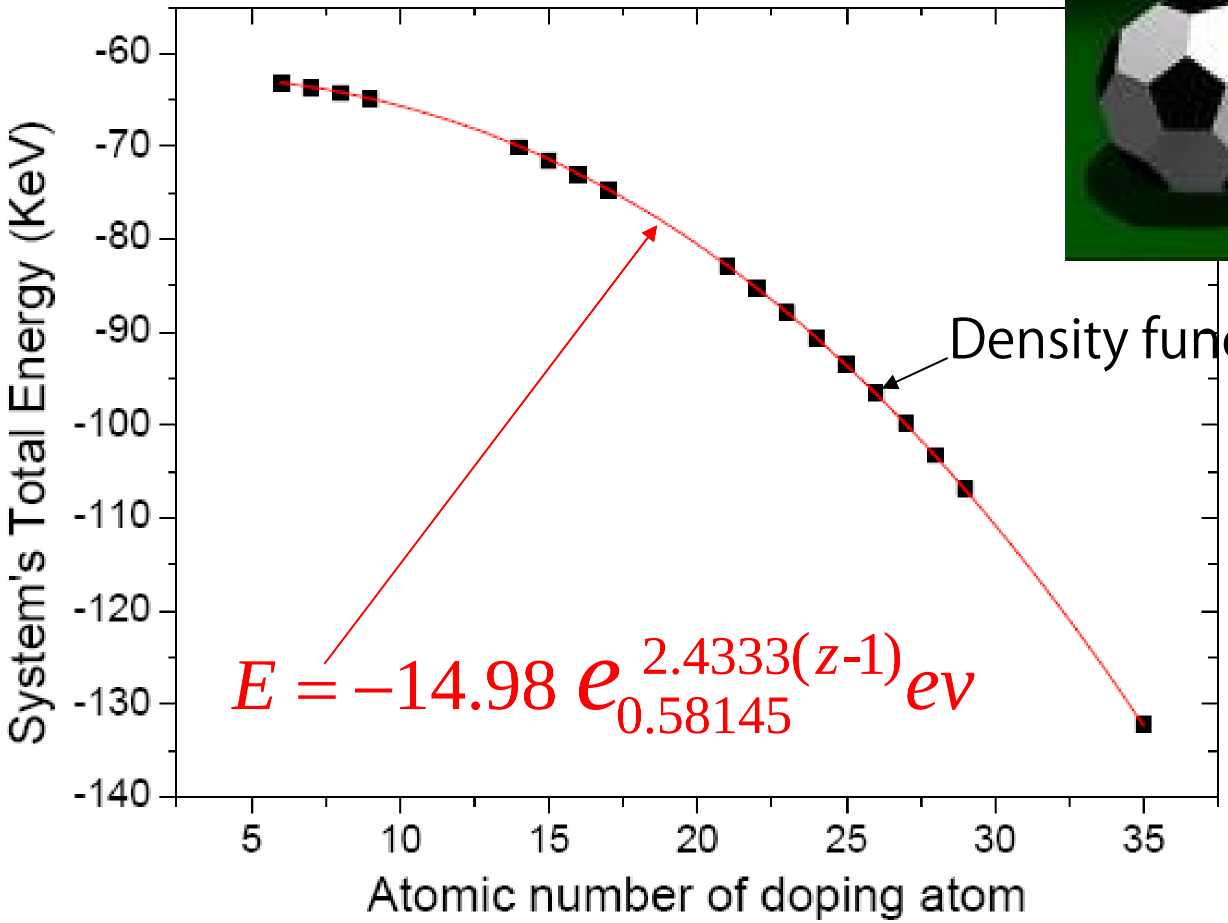
T.S. Biro, G. Purcsel and K. Urmossy Eur Phys J A (June 2009)

MENDELEEV TABLE (Ground state energy)



C.H.S. Amador and L.S. Zambrano (2008)

FULLERENES (C60)

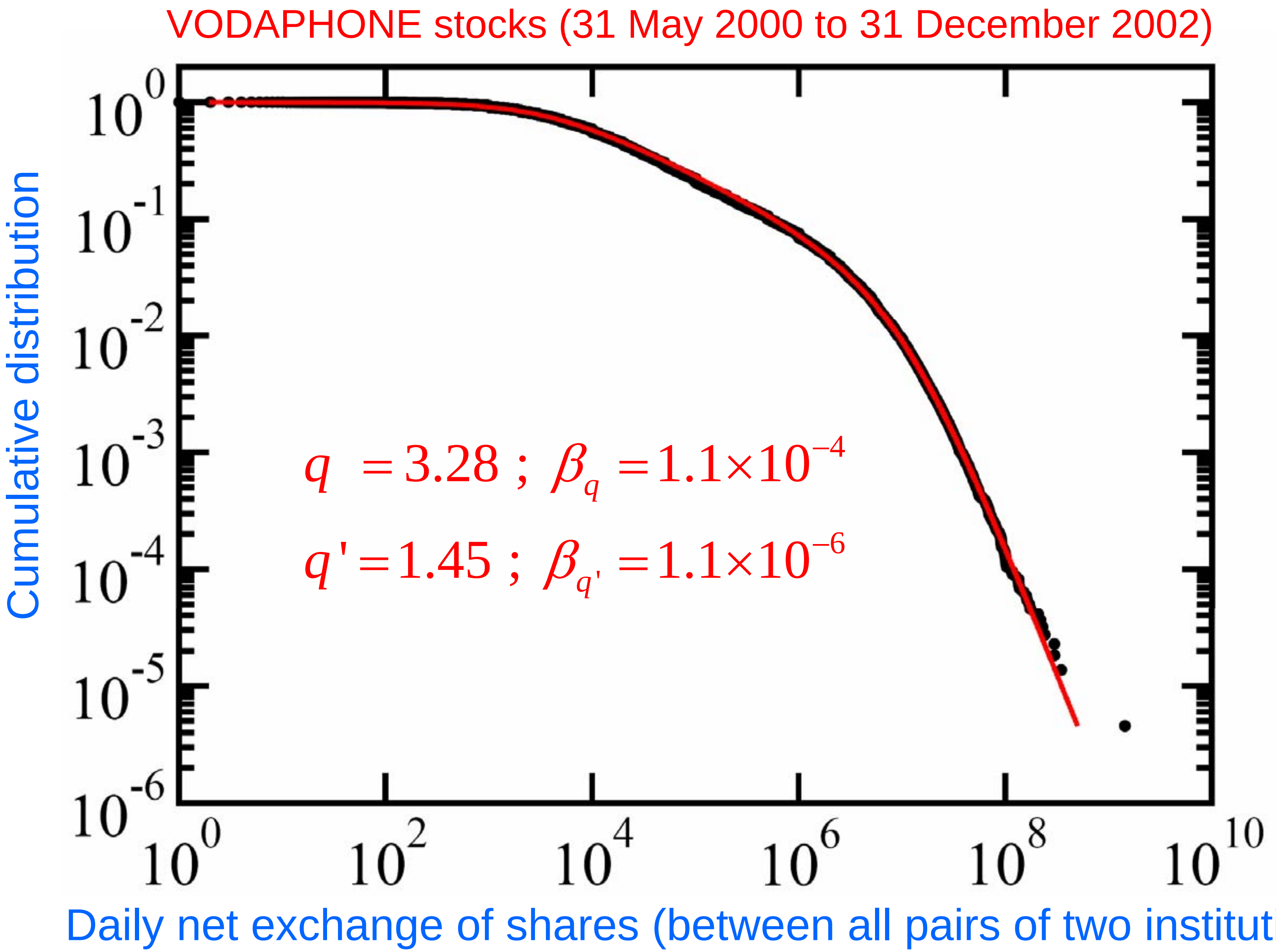


Total energy of C_{60} doped with different atoms trapped at its center as a function of the doping atom atomic number. R^2 = 1 (6-digit precision).

C.H.S. Amador and L.S. Zambrano (2008)

LONDON STOCK EXCHANGE (Block market):

Data: I.I. Zovko; **Fitting**: E.P. Borges (2005)



FACIAL EXPRESSION RECOGNITION USING ADVANCED LOCAL BINARY PATTERNS, TSALLIS ENTROPIES AND GLOBAL APPEARANCE FEATURES

Shu Liao[1,2], Wei Fan[2], Albert C. S. Chung[1,2] and Dit-Yan Yeung[2]

[1]Lo Kwee-Seong Medical Image Analysis Laboratory
and [2]Department of Computer Science and Engineering,
The Hong Kong University of Science and Technology, Hong Kong.

Fig. 4. *Some sample images from the JAFFE database*

[2006 IEEE International Conference on Image Processing, pages 665 – 668]

Features	Classification Accuracy %
AMGFR [15]	82.46
LBP [6]	85.57
ALBP	88.26
Tsallis	85.36
ALBP + Tsallis	91.89
ALBP + Tsallis + NLDAI	94.59

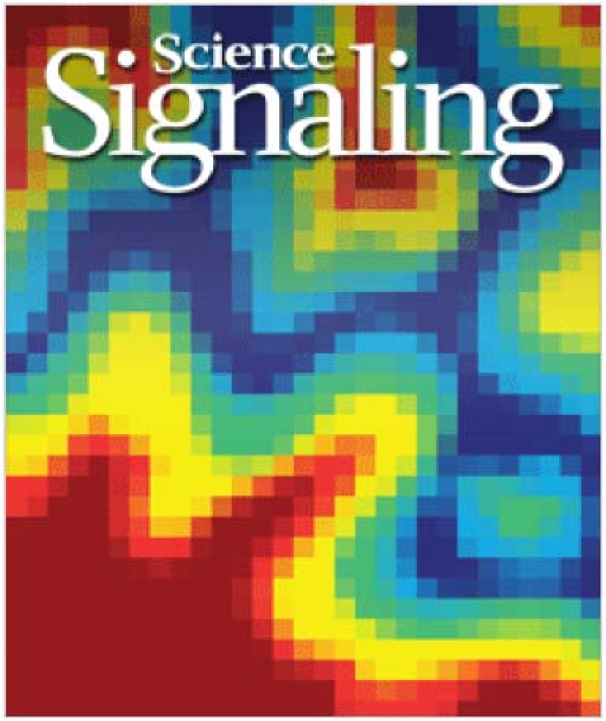
Table 2. *Performance comparison of different approaches with resolution level 64×64 for the images from the JAFFE database*

Features	Classification accuracy (%)		
	48×48	32×32	16×16
AMGFR [15]	78.13	67.83	56.35
LBP [6]	81.44	77.28	68.02
ALBP	84.27	82.74	75.39
Tsallis	79.25	71.04	63.81
ALBP + Tsallis	87.31	85.73	80.40
ALBP + Tsallis + NLDAI	90.54	88.82	84.62

Table 3. *Performance comparison of different approaches with resolution levels 48×48, 32×32 and 16×16 for the images from the JAFFE database*

Science
AAAS

Science Signaling The signal transduction knowledge environment



[1]Department of Chemistry and Biochemistry, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093–0505, USA. [2]San Diego Supercomputer Center, University of California, San Diego, La Jolla, CA 92093–0505, USA. [3]Department of Medicine, School of Medicine, University of California, San Diego, La Jolla, CA 92093–0693, USA. [4]Department of Pediatrics, School of Medicine, University of California, San Diego, La Jolla, CA 92093–0693, USA. [5]Department of Cellular and Molecular Medicine, University of California, San Diego, La Jolla, CA 92093–0693, USA. [6]John and Rebecca Moores UCSD Cancer Center, School of Medicine, University of California, San Diego, La Jolla, CA 92093–0693, USA.

Analysis of Metagene Portraits Reveals Distinct Transitions During Kidney Organogenesis

Igor F. Tsigelny,[1,2*†] Valentina L. Kouznetsova,[3†] Derina E. Sweeney,[3] Wei Wu,[3] Kevin T. Bush,[3] Sanjay K. Nigam[3,4,5,6*]

(Published 9 December 2008; Volume 1 Issue 49 ra16)

Kidney Parameters

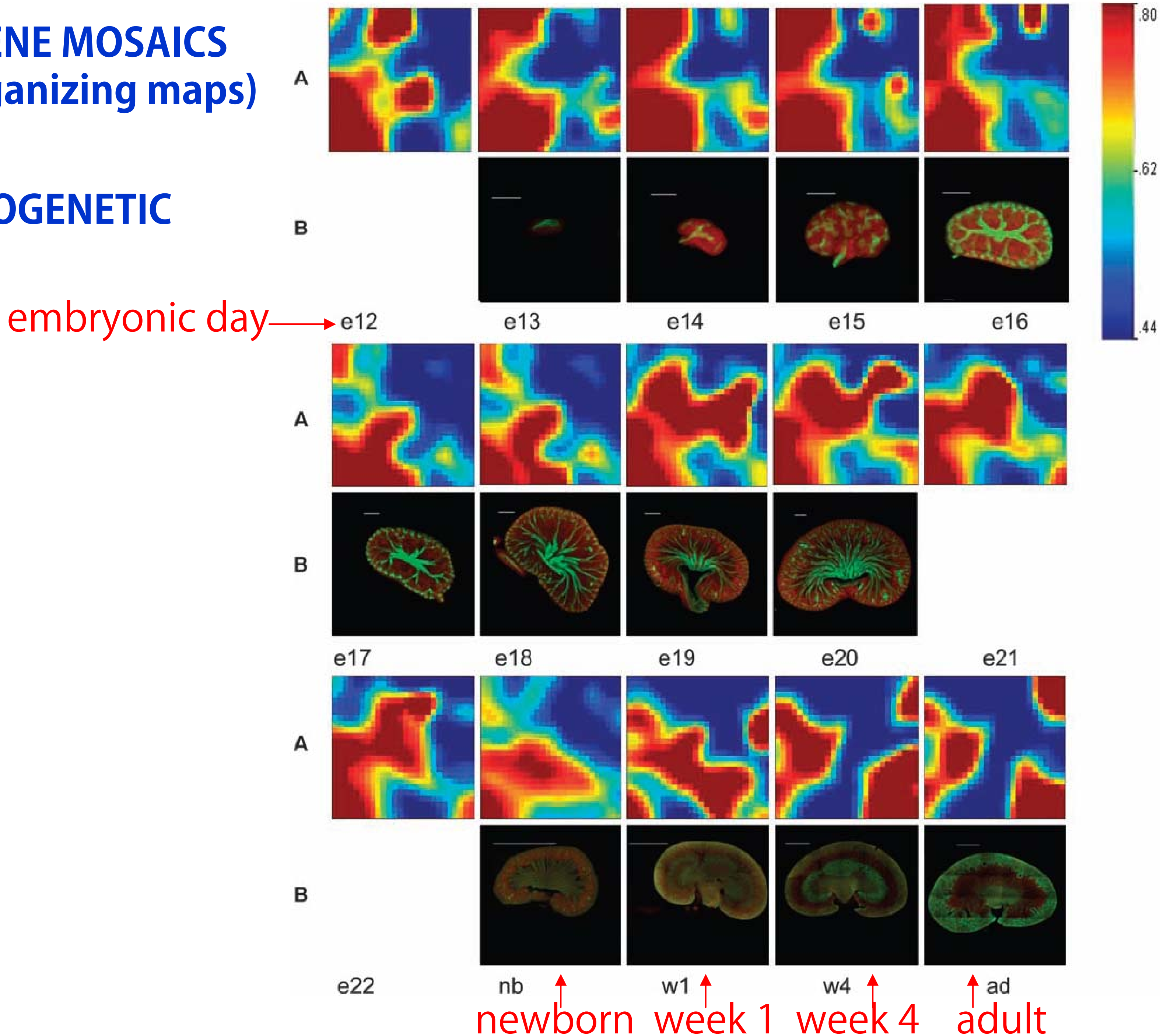
- 20 parameters were measured
  - Embryo kidney mass
  - Embryo mass
  - Kidney/body mass ratio
  - Area
  - DBA stained tissue
  - Perimeter
  - (ellipse) Perimeter (ellipse)
  - Cortex area
  - Medulla area
  - Aspect
  - Major axis
  - Minor axis
  - Feret cortex
  - Feret medulla
  - Feret kidney
  - Roundness
  - Tips
  - Tips per unit
  - Glomeruli
  - Glomeruli per unit

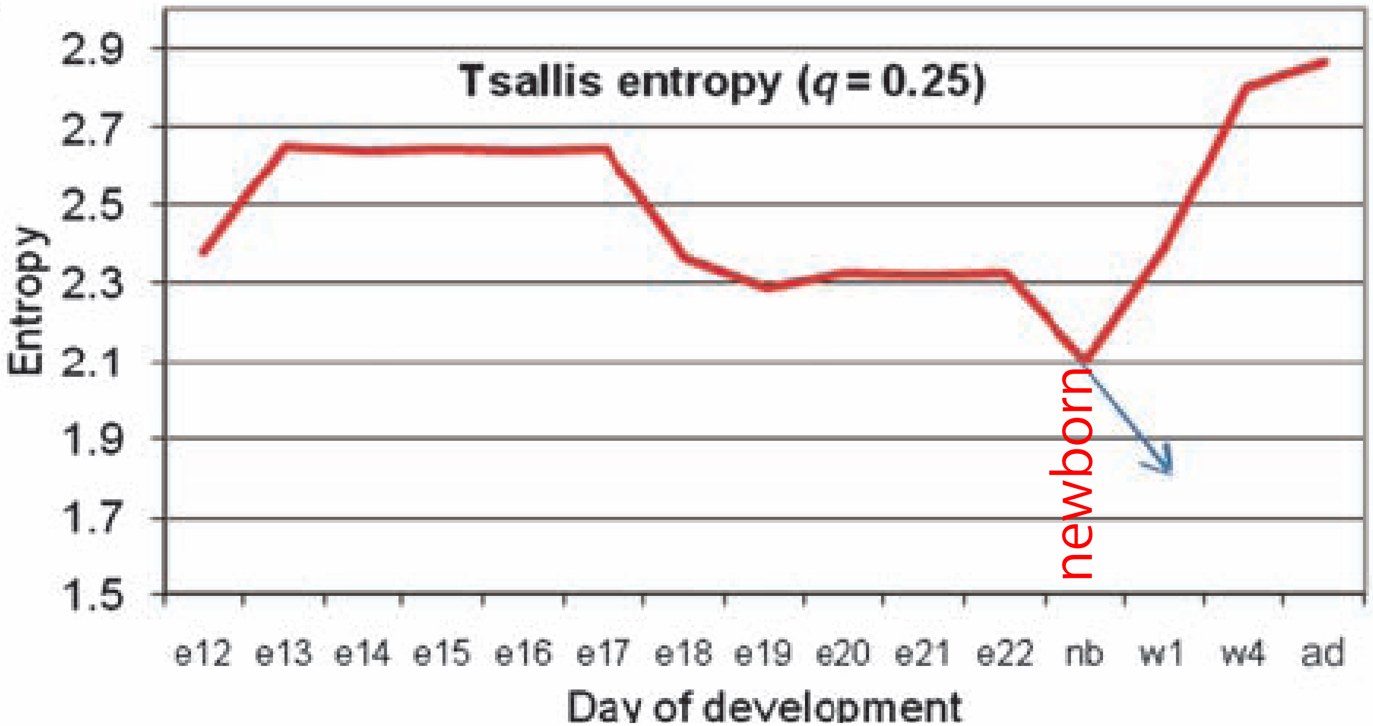
- 1 parameter was calculated
  - Reverse Tsallis entropy

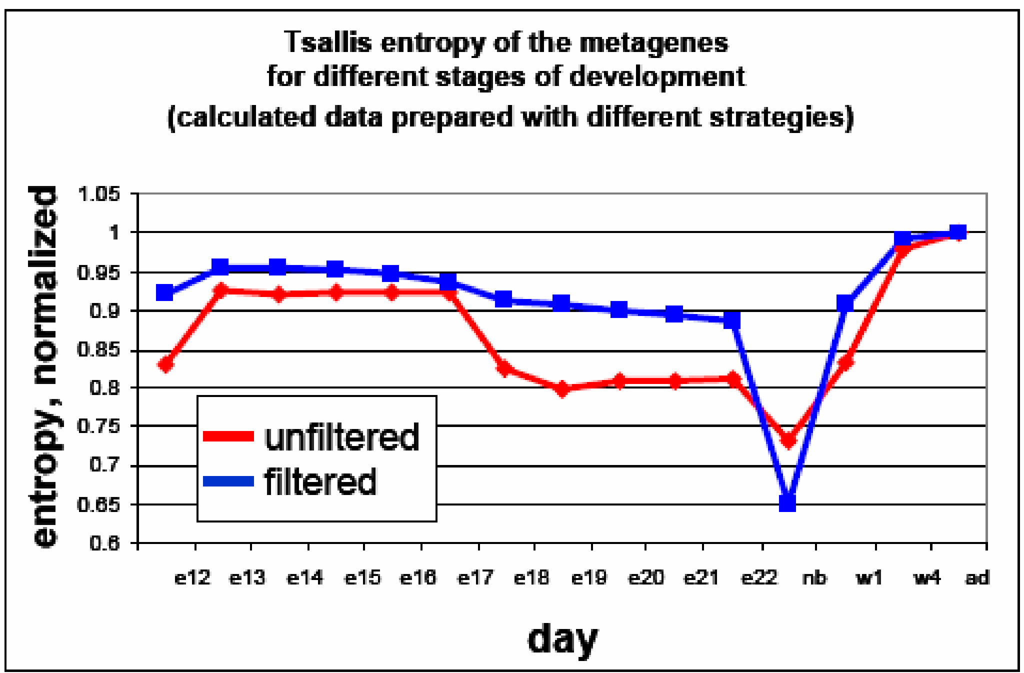
Valentina Kouznetsova
(2/18/2009)

METAGENE MOSAICS (self-organizing maps)

MORPHOGENETIC STAGES




Tsallis entropy (q = 0.25)
Entropy
2.9
2.7
2.5
2.3
2.1
1.9
1.7
1.5
newborn
e12 e13 e14 e15 e16 e17 e18 e19 e20 e21 e22 nb w1 w4 ad
Day of development


Tsallis entropy of the metagenes
for different stages of development
(calculated data prepared with different strategies)
entropy, normalized
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
unfiltered
filtered
e12 e13 e14 e15 e16 e17 e18 e19 e20 e21 e22 nb w1 w4 ad
day

Prediction of the q- triplet: C. T., Physica A **340**,1 (2004)

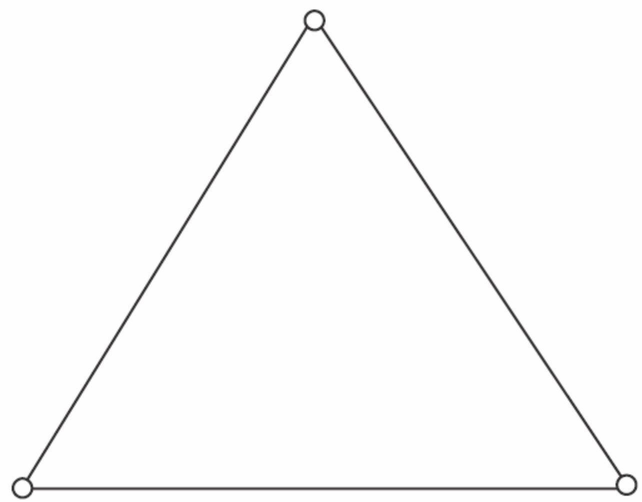


Fig. 2. The triangle of the basic values of q, namely those associated with sensitivity to the initial conditions, relaxation and stationary state. For the most relevant situations we expect $q_{sen} \leqslant 1$, $q_{rel} \geqslant 1$ and $q_{stat} \geqslant 1$. These indices are presumably inter-related since they all descend from the particular dynamical exploration that the system does of its full phase space. For example, for long-range Hamiltonian systems characterized by the decay exponent α and the dimension d, it could be that q_{stat} decreases from a value above unity (e.g., 2 or $\frac{3}{2}$) to unity when α/d increases from zero to unity. For such systems one expects relations like the (particularly simple) $q_{stat} = q_{rel} = 2 - q_{sen}$ or similar ones. In any case, it is clear that, for $\alpha/d > 1$ (i.e., when BG statistics is known to be the correct one), one has $q_{stat} = q_{rel} = q_{sen} = 1$. All the weakly chaotic systems focused on here are expected to have well defined values for q_{sen} and q_{rel}, but only those associated with a Hamiltonian are expected to *also* have a well defined value for q_{stat}.

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Triangle for the entropic index q of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere

L.F. Burlaga*, A.F. -Viñas

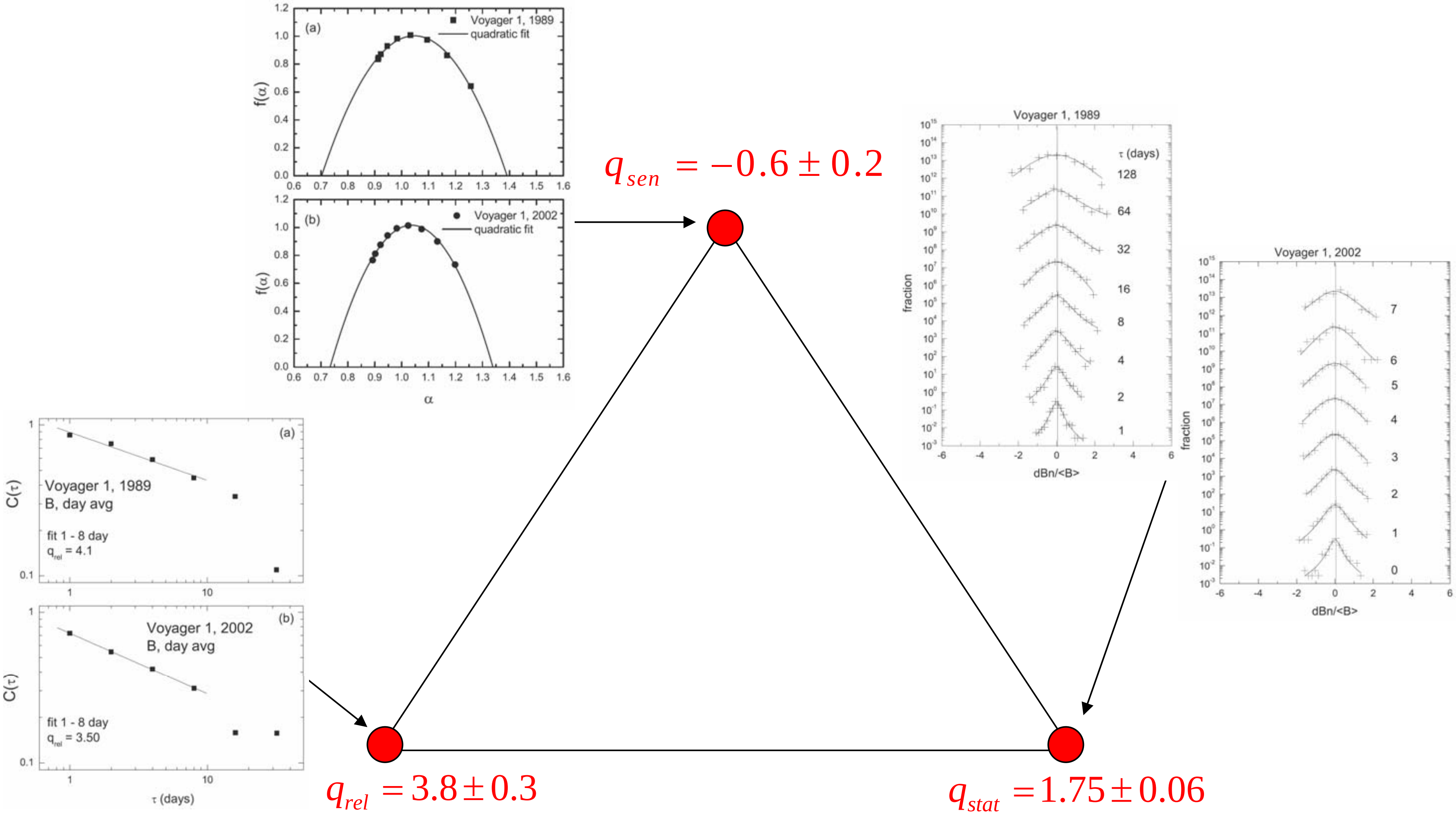
Laboratory for Solar and Space Physics, Code 612.2, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA

Received 10 June 2005
Available online 11 July 2005

SOLAR WIND: Magnetic Field Strength

L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]



Playing with additive duality $(q \to 2-q)$

and with multiplicative duality $(q \to 1/q)$

(*and using numerical results related to the $q-$generalized central limit theorem*)

we conjecture

$$q_{rel} + \frac{1}{q_{sen}} = 2 \qquad \textit{and} \qquad q_{stat} + \frac{1}{q_{rel}} = 2$$

hence $$1 - q_{sen} = \frac{1-q_{stat}}{3-2\,q_{stat}}$$

hence only one independent!

Burlaga and Vinas (NASA) most precise value of the $q-$triplet is

$$q_{stat} = 1.75 = 7/4$$

hence $q_{sen} = -\,0.5 = -\,1/2$ (*consistent with* $q_{sen} = -0.6 \pm 0.2$!)

and $q_{rel} = 4$ (*consistent with* $q_{rel} = 3.8 \pm 0.3$!)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

$$\varepsilon_{sen} \equiv 1 - q_{sen} = 1 - (-1/2) = 3/2$$

$$\varepsilon_{rel} \equiv 1 - q_{rel} = 1 - 4 = -3$$

$$\varepsilon_{stat} \equiv 1 - q_{stat} = 1 - 7/4 = -3/4$$

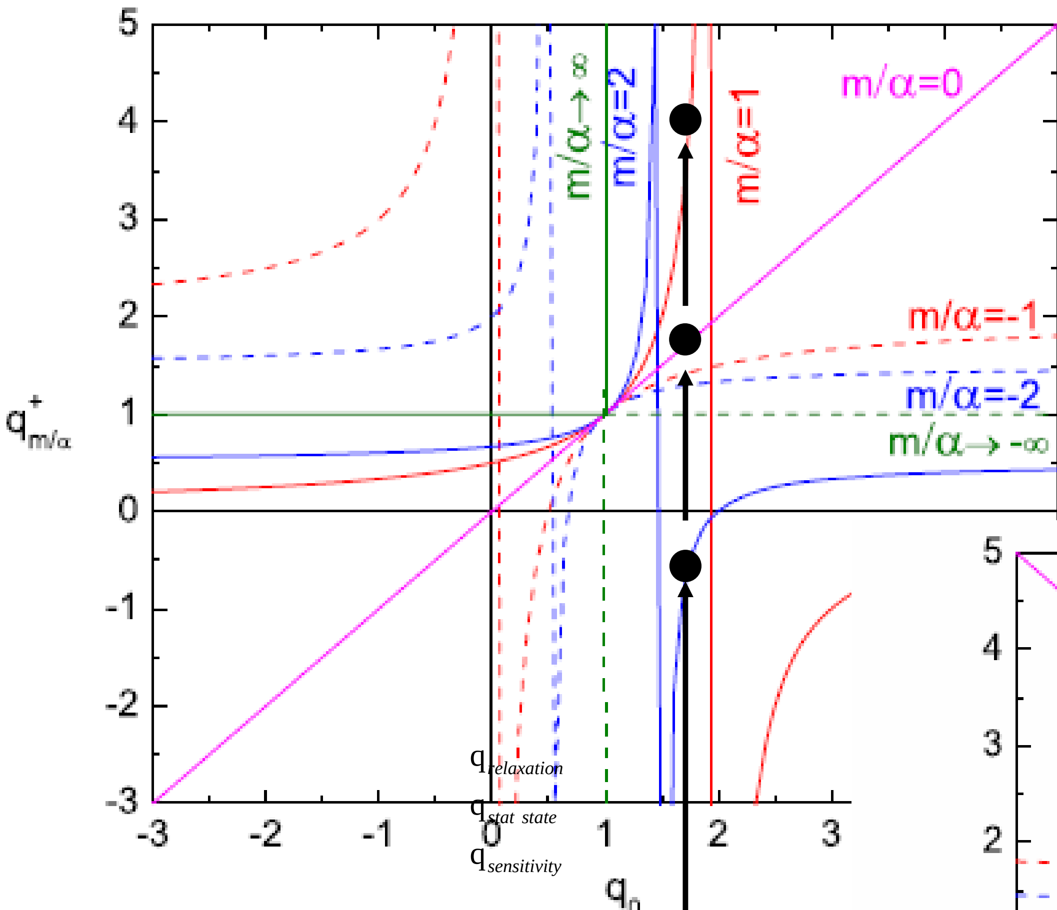
We verify

$$\varepsilon_{stat} = \frac{\varepsilon_{sen} + \varepsilon_{rel}}{2}$$ (arithmetic mean!)

$$\varepsilon_{sen} = \sqrt{\varepsilon_{stat} \ \varepsilon_{rel}}$$ (geometric mean!)

$$\varepsilon_{rel}^{-1} = \frac{\varepsilon_{sen}^{-1} + \varepsilon_{stat}^{-1}}{2}$$ (harmonic mean!)

N.O. Baella (2008)



$$\frac{1}{1-q^{+}_{m/\alpha}}=\frac{1}{1-q_0}+\frac{m}{\alpha}$$

$$\frac{1}{1-q^{-}_{m/\alpha}}=\frac{1}{q_0-1}+\frac{m}{\alpha}$$

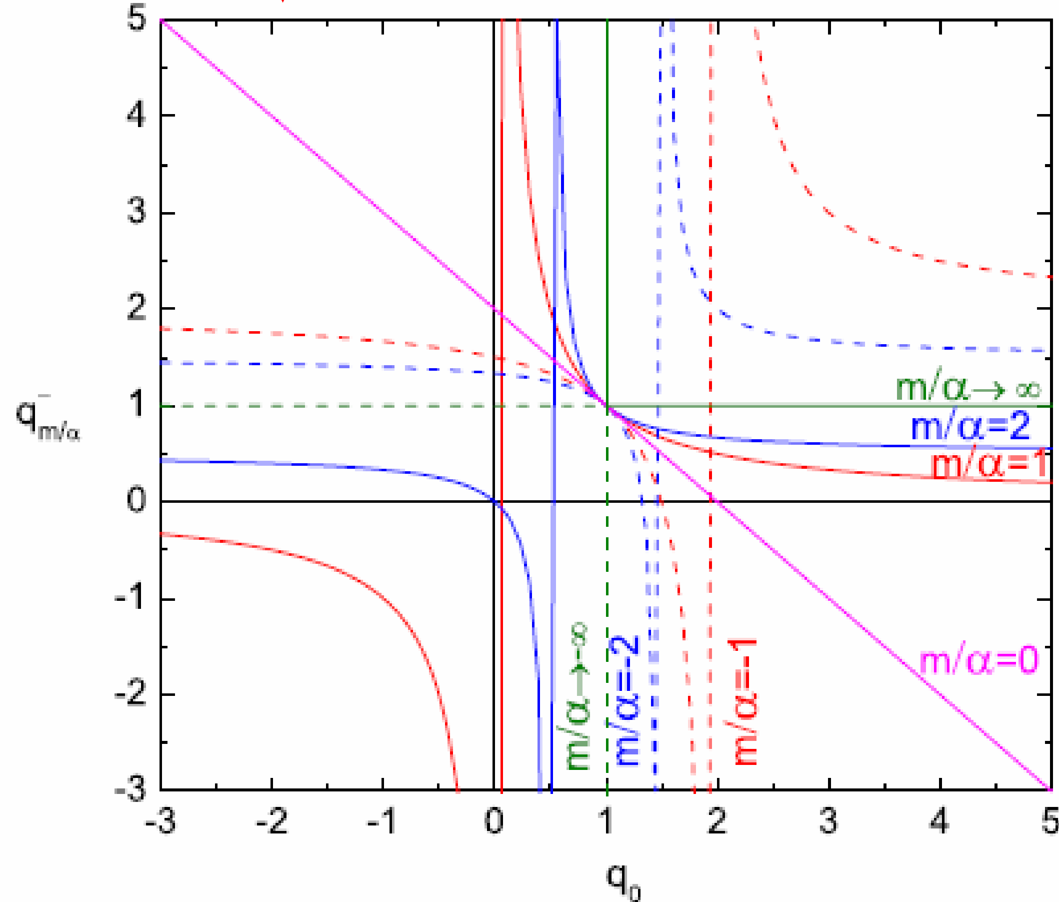
$$(0<\alpha\leq 2;\ m=0,\pm 1,\pm 2,\ldots)$$

solar wind
q-triplet

$q_{relaxation}$

$q_{stat\ state}$

$q_{sensitivity}$



C. T., Braz J Phys (2009), in press

Strongly non-Markovian noise → Nonlinear Fokker-Planck equation: A mesoscopic mechanism leading to nonextensive statistical mechanics

$$\frac{\partial\, p(x,t)}{\partial\, t} = \frac{\partial}{\partial\, x}\left[\frac{\partial U(x)}{\partial\, x}\, p(x,t)\right] + D\,\frac{\partial^2 [p(x,t)]^{2-q}}{\partial\, x^2} \quad [q<3;\ D(2-q)>0;\ t\ge 0]$$

Plastino and Plastino, Physica A **222** (1995) 347; Fuentes and Caceres, PLA **372** (2008)

1) The imposition of the H-theorem in this equation mandates the entropy to be S_q

Schwammle, Curado and Nobre,
Eur Phys J B **58** (2007) 159; Phys Rev E **76** (2007) 041123

2) Stationary state in the presence of any confining potential $U(x)$:

$$p(x,\infty) \propto e_q^{-\beta\,[U(x)-U(0)]}\ ,\ \text{where } \beta>0$$

3) If $\quad U(x) = -k_1 x + \frac{1}{2}k_2 x^2 \quad (k_2>0),$ then

$$p(x,t) \propto e_q^{-\beta(t)\,x^2},\ \text{where } 0<\beta(\infty)<\infty$$

C T and DJ Bukman, Phys Rev E **54** (1996) R2197

4) If $\quad U(x)=0,$ then $\quad p(x,t) \propto e_q^{-\beta(t)\,x^2}$, where $\beta(t) \propto 1/t^{\frac{2}{3-q}}$,

hence x^2 scales like t^γ with $\gamma = \dfrac{2}{3-q}$ **(prediction)**

Multiplicative noise → Linear inhomogeneous Fokker-Planck equation: Another mesoscopic mechanism leading to nonextensive statistical mechanics

C. Anteneodo and C. T., J Math Phys **44** (2003) 5194

$$\frac{dx}{dt} = f(x) + g(x)\ \xi(t) + \eta(t)$$

where $\xi(t)$ and $\eta(t)$ are independent zero-mean white Gaussian noises with amplitudes M and A. It follows

$$\frac{\partial\, p(x,t)}{\partial\, t} = -\frac{\partial\, \left[f(x)\, p(x,t) \right]}{\partial\, x} + M \frac{\partial}{\partial x}\left\{ g(x) \frac{\partial [g(x)\, p(x,t)]}{\partial x} \right\} + A\ \frac{\partial^2 \left[p(x,t) \right]}{\partial\, x^2}$$

If the deterministic drift is proportional to the multiplicative-noise induced drift, i.e., if $f(x) = -\tau\ g(x)\ g'(x)$, [$e.g.,\ f(x) \propto g(x) \propto x$]

natural first physical choice

then the stationary state is given by

$$p(x,\infty) \propto e_q^{-\beta\ [g(x)]^2}$$

with $q = \dfrac{\tau + 3M}{\tau + M} \geq 1$ and $\beta \equiv \dfrac{1}{kT} = \dfrac{\tau + M}{2A}$

[see also L. Borland, Phys Lett A **245** (1998) 67]

$q = 1$ statistical mechanics :

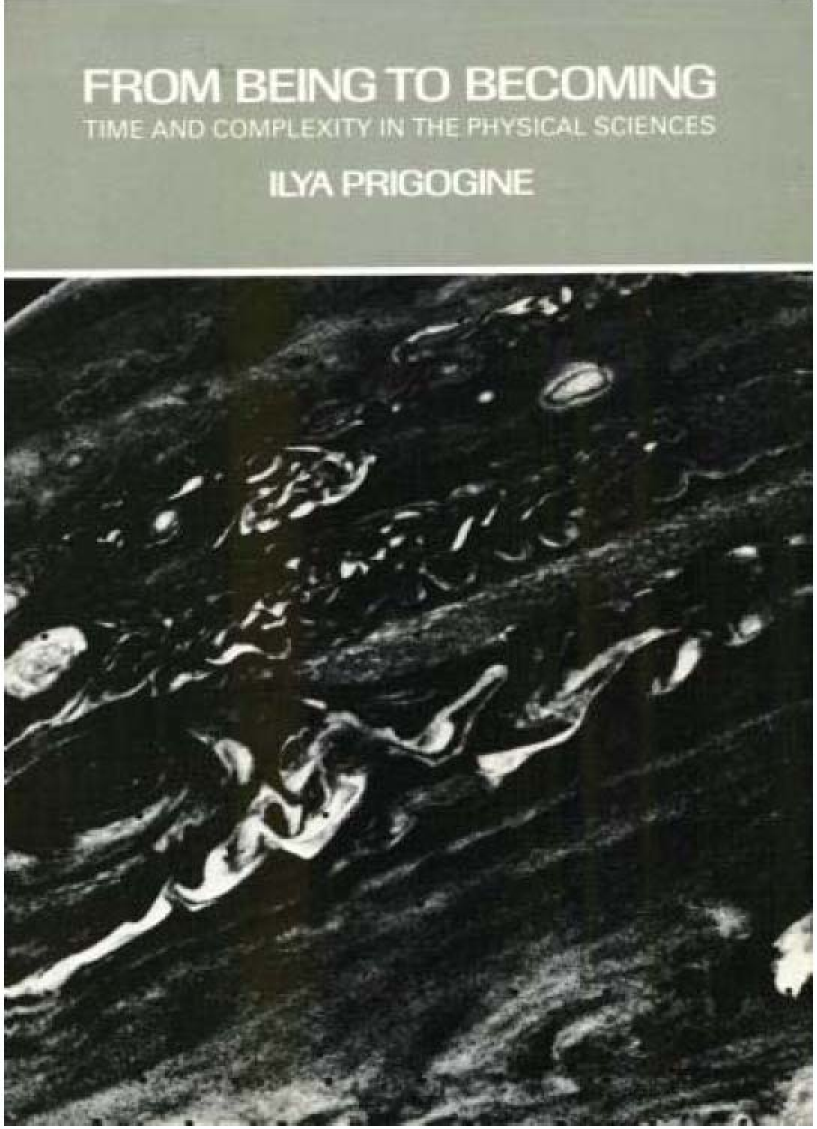
Non, je ne regrette rien…
baleyé, oublié, je me fous du passé…

Edith Piaf

$q \neq 1$ statistical mechanics :

Je me souviens

Québec



Statistical mechanics of *BEING* → Boltzmann-Gibbs (q = 1)

Statistical mechanics of *BECOMING* → Nonextensive statistical mechanics

Introduction to Nonextensive Statistical Mechanics

APPROACHING A COMPLEX WORLD

Constantino Tsallis

Springer

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

2734 articles by 2117 scientists from 64 countries

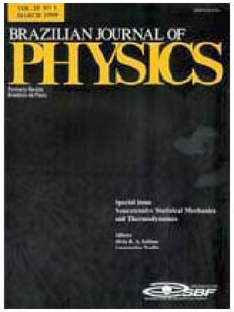
[27 May 2009]

CONTRIBUTORS **(2734 MANUSCRIPTS)** [Updated 27 May 2009]

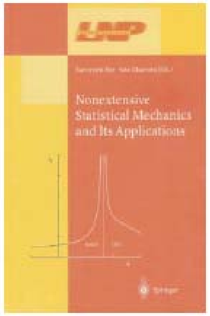
BRAZIL	382	BELGIUM	15	LITHUANIA	3
USA	302	SOUTH KOREA	14	SLOVAK	3
CHINA	165	ISRAEL	13	PUERTO RICO	3
JAPAN	146	NETHERLANDS	13	BOLIVIA	2
ITALY	130	HUNGARY	12	CZECK	2
FRANCE	122	IRAN	12	FINLAND	2
SPAIN	86	DENMARK	11	KAZAKSTAN	2
ARGENTINA	86	SOUTH AFRICA	11	MOLDOVA	2
UNIT. KINGDOM	76	CUBA	10	PHILIPINES	2
GERMANY	65	CHILE	7	ARMENIA	1
POLAND	48	VENEZUELA	7	COLOMBIA	1
RUSSIA	44	ROMENIA	7	CYPRUS	1
INDIA	41	NORWAY	5	INDONESIA	1
CANADA	36	SINGAPORE	5	JORDAN	1
TURKEY	33	CROATIA	4	MALAYSIA	1
GREECE	33	EGYPT	4	SAUDI ARABIA	1
UKRAINE	27	SLOVENIA	4	SERBIA	1
AUSTRIA	24	SWEDEN	4	SRI LANKA	1
MEXICO	20	TAIWAN	4	THAILAND	1
PORTUGAL	19	URUGUAY	4	UZBEKISTAN	1
SWITZERLAND	17	IRELAND	4		
AUSTRALIA	15	BULGARIA	3		

64 COUNTRIES 2117 SCIENTISTS

NONEXTENSIVE STATISTICAL MECHANICS AND THERMODYNAMICS



Nonextensive Statistical Mechanics and Thermodynamics, SRA Salinas and C Tsallis, eds, Brazilian Journal of Physics **29**, Number 1 (**1999**)



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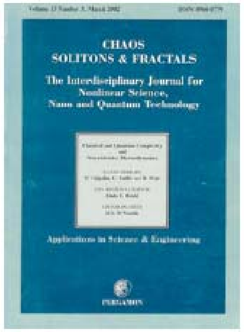
Non Extensive Thermodynamics and Physical Applications, G Kaniadakis, M Lissia and A Rapisarda, eds, Physica A **305**, Issue 1/2 (**2002**)



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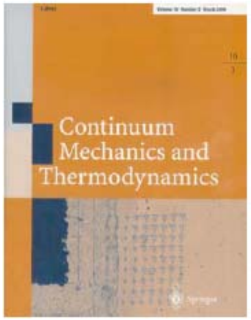
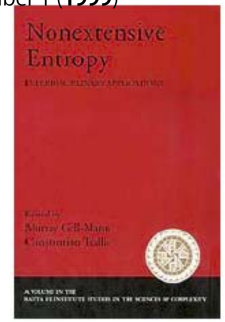
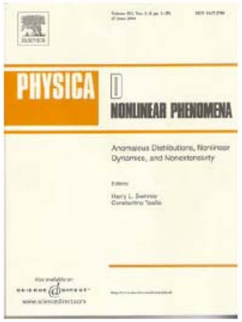


Nonadditive Entropy and Nonextensive Statistical Mechanics, M Sugiyama, ed, Continuum Mechanics and Thermodynamics **16** (Springer, Heidelberg, **2004**)



Nonextensive Entropy - Interdisciplinary Applications, M Gell-Mann and C Tsallis, eds, (Oxford University Press, New York, **2004**)



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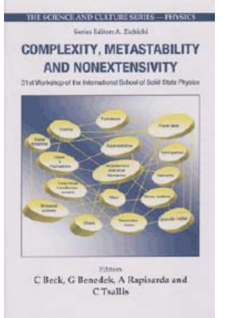
HL Swinney and C Tsallis, eds, Physica D **193**, Issue 1-4 (**2004**)

News and Expectations in Thermostatistics
G Kaniadakis and M Lissia, eds Physica A **340**, Issue 1/3 (**2004**)

Trends and Perspectives in Extensive and Non-Extensive Statistical Mechanics H Herrmann, M Barbosa and E Curado, eds, Physica A **344**, Issue 3/4 (**2004**)



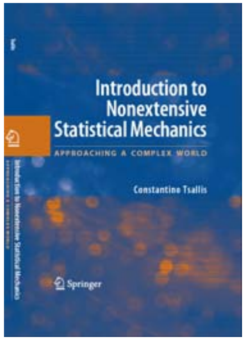
Complexity, Metastability and Nonextensivity, C Beck, G Benedek, Rapisarda and C Tsallis, eds, (World Scientific, Singapore, **2005**)

Nonextensive Statistical Mechanics: New Trends, New Perspectives, JP Boon and C Tsallis, eds, Europhysics News (European Physical Society, **2005**)

Fundamental Problems of Modern Statistical Mechanics, G Kaniadakis, A Carbone and M Lissia, eds, Physica A **365**, Issue 1 (**2006**)

Complexity and Nonextensivity: New Trends in Statistical Mechanics, S Abe, M Sakagami and N Suzuki, eds, Progr. Theoretical Physics Suppl **162** (**2006**)

Complexity, Metastability and Nonextensivity, S Abe, H Herrmann, P. Quarati, A Rapisarda and C Tsallis, eds, American Institute of Physics Conference Proc. **965** (New York, **2007**)



Introduction to Nonextensive Statistical Mechanics - Approaching a Complex World, C. Tsallis (Springer, 2009)

Pois é, mal dá para acreditar, mas já se passaram mais de 20 anos da proposta de generalização, baseada na entropia não aditiva Sq, da magnífica mecânica estatística de Boltzmann-Gibbs [C. T., J. Stat. Phys. **52**, 479 (1988)]. E ainda tem inúmeros, fascinantes e complexos aspectos a serem esclarecidos. Mas vários outros estão já razoavelmente entendidos. Uma revisão breve será apresentada, com ênfase nas diversas verificações -- experimentais, observacionais e computacionais – atualmente disponíveis das predições da teoria.

Bibliografia: (i) http://tsallis.cat.cbpf.br/biblio.htm ; (ii) C. Tsallis, *Introduction to Nonextensive Statistical Mechanics – Approaching a Complex World* (Springer, New York, 2009).